# Advertising for Attention in a Consumer Search Model*

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**Abstract**

We model the idea that when consumers search for products, they first visit the firms whose advertising is more salient. Since the gains a firm derives from being visited earlier than the rivals increase in search costs, investments in advertising go up as search costs rise. In spite of the standard price effects of higher search costs, we find that firms may end up obtaining lower profits when search costs increase. In our model, firms engage in an advertising battle to raise consumer attention and not to lose market shares. The game has the features of a prisoners' dilemma-like situation so if advertising were banned (or if firms could agree not to advertise), firms would be better off and welfare would increase. We extend the basic model by allowing for firm heterogeneity. It turns out that those firms whose advertising is intrinsically more salient and therefore happen to raise attention more frequently charge lower prices in equilibrium and obtain higher profits. As advertising cost asymmetries increase, aggregate profits increase, advertising outlays fall and welfare increases.

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# 1 Introduction

As shown by its importance in terms of GDP, advertising has since long been an important aspect of economic activity. For example, in the US advertising expenditures constitute around 2.2% of GDP. In the era of the new communication technologies (cable and satellite TV and radio stations, specialized magazines, classified Internet homepages, electronic newsgroups, etc.), world spending in advertising has grown significantly. According to PriceWaterhouseCoopers worldwide advertising in 2005 amounted to a staggering $385 billion (PriceWaterhouseCoopers, 2005). This amount is set to grow to over half-a-trillion dollars in 2010.

Economists have dedicated a significant amount of effort to understand the role of advertising in markets. Traditionally, advertising has been thought of as a sunk cost firms incur with the purpose of enhancing consumers' willingness-to-pay for their products. This type of advertising has been termed *persuasive* advertising. The idea behind its utilization is that persuasive advertising creates fictitious product differentiation, which softens price competition and increases firm profits (Kaldor, 1950; Galbraith, 1967; Solow 1967).

Since Telser (1964) and Nelson (1970, 1974), the view of advertising as a device to transmit information has been gradually gaining support by economists. Generally speaking, *informative* advertising helps to reduce the lack of information existing in product markets by, for instance, communicating firms' existence, their prices, qualities or locations. Informative advertising increases the information economic agents possess, which increases competition between firms and raises social welfare.[1]

It would be hard to fit all of the advertising we observe in the real world into the informative advertising models studied so far in the Industrial Organization literature for various reasons. For example, consumer needs and advertising campaigns are not always contemporaneous; by the time a consumer decides to search for a product the price announced in the ads has expired. Often, we see advertisements conveying little direct information and many ads do not feature the prices of the products. One theory that explains the existence of the latter sorts of advertisement is based on the idea that advertising functions as a signal of quality (Nelson, 1974; Kihlstrom and Riordan, 1984; and Milgrom and Roberts, 1986). According to this view, when quality cannot be observed in advance to a purchase, advertising can still inform consumers about the quality of a product indirectly. This theory, however, does not explain that part of the advertising concerning horizontally differentiated products for which quality differences are negligible.

In this paper we propose a novel model where the advertisements of a firm compete for the attention of consumers who search around for satisfactory products. In many occasions the main role of advertisements is to let consumers know the array of products that a consumer can find in its store and this is the role we emphasize in this paper. More specifically, we

[1] See e.g. the papers of Bester and Petrakis (1995), Butters (1977), Grossman and Shapiro (1984), Shapiro (1980) and Stahl (1994).

model the following market process. Suppose that a consumer wants to purchase a certain product, but is not sure in which shop she can find the nicest (in her view) version of the product. After some thinking, she recalls an advertisement from a shop selling the kind of product she is interested in and goes there to see what exactly the shop has on offer and the price it charges. Such a visit is costly. If the product is not to her liking, she has the option to walk away from the shop, try to recall an alternative vendor and visit such firm, again at some positive search cost. This process continues until the consumer finds a deal that is so attractive that searching further is not worth her while, or after she has visited all the firms in the market. In the latter case, she returns to the firm that offers her the best deal.

This model constitutes a suitable framework to understand the effects of search costs on advertising, prices and profits. Since search costs are known to have far-reaching effects on prices and profits, it is clear that the magnitude of search costs influences the gains a firm derives from being visited earlier than rival firms. As search costs rise, *a priori*, consumer awareness becomes more valuable for firms because consumers are likely to visit salient firms more frequently, or earlier, than other less salient rivals.[2]

If the firms did not advertise to raise consumer attention, a consumer would recall a given firm with the same probability as any other firm. To model the idea that firms visit first the most salient firms, we assume that the probability that a consumer recalls a shop at every stage in the search process is proportional to that shop's share in total industry advertising expenditures. This modelling of the recall process in inspired from Comanor and Wilson (1974, pg. 47):

> *To the extent that the advertising of others creates 'noise' in the market, one must 'shout' louder to be heard, so that the effectiveness of each advertising message declines as the aggregate volume of industry advertising increases.*

In other words, as the amount of advertising grows, it is increasingly difficult for a firm to stand out from the crowd, to get the attention of consumers, and to convince them to come to its point-of-sale and try its product. In his extensive literature survey on advertising, Bagwell (2007, pg. 1798) argues that this approach of advertising warrants formalization:

> *Future work might revisit this noise effect, in a model that endogenizes the manner in which consumers with finite information-storage capabilities manage (as possible) their exposure to advertising".*

Our analysis reveals that both price and advertising expenditures are increasing in search costs. When searching around for a satisfactory product becomes more costly, a typical firm has more market power over each consumer that pays it a visit and therefore the firm can safely raise its price. At the same time, when search costs increase it becomes more appealing for a firm to invest in saliency because once consumers show up at the shop to inspect its

[2]For an empirical study on how the "salience" of one brand inhibits the recall of alternative brands see Alba and Chattopadhyay (1986).

product, it is less likely that they decline the offer and walk away to search for another product.

The effect of an increase in search costs on firm profitability is ambiguous. On the one hand, with greater search costs firms can charge higher prices but on the other hand firms advertise more. If search costs are small, the price effect dominates and equilibrium profits increase with raising search costs. By contrast, when search costs are initially high and go up even further the gains associated to the price effect can be more than offset by the rent-dissipation effect of increasing advertising expenditures. This result provides an instance in which firms do not necessarily benefit from higher search costs. In fact, when search costs are large and advertising costs relatively low, we find that firms profits are lower than what they would be in a frictionless world with no search costs at all.

In our (basic) model firms have symmetric advertising technologies and firms find themselves in a classic prisoners' dilemma situation. Out of equilibrium, a firm gains by slightly undercutting the rivals and increasing its advertising effort thereby becoming more salient and so attracting a larger share of the consumers' first-visits. However, in symmetric equilibrium, all firms advertise with the same intensity, which implies that a consumer ends up recalling every firm with the same probability and therefore advertising has no price effects. In this situation, the firms would be better off if advertising were banned. From a social welfare point of view, advertising is purely wasteful.

In an extension of the basic model, we study competition among two firms that employ different advertising technologies. In equilibrium, the most advertising-efficient firm advertises more intensively and so attracts a larger share of the consumer first-visits. It turns out that pricing behavior consistent with these advertising expenditures requires the most advertising-efficient firm to charge a lower price than the less advertising-efficient counterpart. The intuition behind this result is as follows. In equilibrium, each firm receives a share of the consumer first-visits and second-visits. First-visits are more price-sensitive than second-visits because the fact that consumers have walked away from a given shop to visit another shop indicates that the first vendor did not have a satisfactory product. Thus, the pricing result arises because, relative to the most efficient firm's pool of consumers, the less-efficient firm receives a larger share of less price-sensitive consumers. Despite charging a lower price and advertising more intensively, the most advertising-efficient firm turns out to obtain greater profits than the less efficient rival. Since the firm that advertises more offers a better deal to consumers, it turns out to be individually rational for a consumer to stick to the recall process we postulate in our model.

We study the comparative statics of larger advertising cost asymmetries. In this case where firms are asymmetric, advertising has social value because it helps channel consumers first-visits towards better deals. In fact, as advertising cost asymmetries increase more consumers go first to the cheapest store and fewer consumers shop around. Consumer surplus however decreases because of the price increase of the less advertising-efficient firm. Savings in advertising and search costs outweigh the consumer losses and overall welfare goes up as advertising cost asymmetries rise.

Our paper is a contribution to the relatively small body of work at the intersection of the search and advertising literatures.[3] Put in the perspective of the search literature, we present a search model where the order in which firms are visited is influenced by advertising efforts. The search literature is fairly extensive but has generally assumed that consumers sample the firms with the same probability. One of the reasons is that the bulk of the search literature has used equilibrium models with infinitely many firms.[4] One exception is the paper of Arbatskaya (2007), who studies a market for homogeneous products where the order in which firms are visited is exogenously given. In equilibrium prices must fall as the consumer walks away from the firms visited first. In our model the order of search is determined endogenously; moreover, the fact that firms sell horizontally differentiated products implies that prices increase in the order in which firms are sampled. In an empirical study, Hortaçsu and Syverson (2004) also present a model where sampling probability variation across firms is used to explain price dispersion in the mutual funds industry. Hortaçsu and Syverson use, among other variables, advertising outlays as a proxy for the probabilities a fund is sampled in the market. Wilson (2008) also studies a model where the order of search is endogenous. In his model firms can choose the magnitude of the search costs consumers need to incur to conduct a transaction. Consumers visit first the low search cost firm and, like in Arbatskaya (2007), prices fall in the search order. Finally, and more related to the specific model we study here, Armstrong et al. (2007) study the implications of "prominence" in search markets. In their model, there is a firm that is always visited first and this firm charges lower prices and derives greater profits than the rest of the firms, which are sampled randomly after consumers have visited the prominent firm. Our model can be interpreted as one in which firms invest in prominence but where saliency can only be imperfect.

At the industry level, our model predicts a positive correlation between advertising expenditures and prices. This is somewhat in line with the prediction from the signalling literature mentioned above where high prices and high advertising expenditures signal high quality (Milgrom and Roberts, 1986). Within an industry, our model predicts that firms that advertise more charge lower prices and have larger market shares. This is consistent with results from the literature on informative advertising since here too advertising is ultimately directing consumers towards cheaper prices. Stahl (1994) studies a market for homogeneous products where firms engage in advertising to attract consumers. In equilibrium sellers choose a common advertising level and mix over prices so there is no correlation between advertising intensities and price levels. Robert and Stahl (1993) study the same sort of advertising market where consumers can also search. Their model has predictions similar to

[3]The advertising and search literatures have followed separate paths. The typical advertising model assumes the cost of search to be sufficiently large that consumers not informed by the advertisements never initiate search. Likewise, the typical search model assumes the cost of advertising to be so large that advertising never occurs. Notable exceptions are Robert and Stahl (1993) and Janssen and Non (2008).

[4]See the seminal contributions of, among others, Rob (1985), Bénabou (1993), Burdett and Judd (1983), Carlson and McAfee (1983), Stahl (1989), Reinganum (1979) and Wolinsky (1986). More recent work includes Anderson and Renault (1999), Janssen and Moraga-González (2004, 2007), Janssen et al. (2005) and Rauh (2004, 2007).

ours but for different reasons. In their paper lower prices are advertised more intensively. Moreover, like in our paper advertising levels converge to zero as the search cost goes to zero. In our model the existence of search costs creates a market for attention while in their paper search costs lead to a market for price advertising. As search costs converge to zero, the *raison-d'tre* of advertising markets vanishes.[5]

The remainder of this paper is structured as follows. In section 2 we describe the set-up of the model. The equilibrium results for symmetric firms are derived in subsection 3.1, and the results on the effects of search costs on advertising efforts, prices and profits are given in subsection 3.2. Section 4 presents results for a market with asymmetric firms. Section 5 concludes. All proofs are gathered in an Appendix at the end of the paper.

# 2 The model

On the supply side of the market there are $n$ firms selling horizontally differentiated products. They employ a constant returns to scale technology of production and we normalize unit production costs to zero. On the demand side of the market, there is a unit mass of consumers. A consumer $m$ has tastes described by an indirect utility function

$$u^{mi}(p_i) = \varepsilon_{mi} - p_i,$$

if she buys product $i$ at price $p_i$. The parameter $\varepsilon_{mi}$ can be thought of as a match value between consumer $m$ and product $i$. Match values are independently distributed across consumers and products. We assume that the value $\varepsilon_{mi}$ is the realization of a random variable with distribution $F$ and a continuously differentiable density $f$ with support that is normalized to $[0, 1]$. No firm can observe $\varepsilon_{mi}$ so practising price discrimination is not feasible. Let $p^m$ denote the monopoly price, i.e., $p^m \equiv \arg\max_p\{p(1 - F(p))\}$.

A consumer must incur a search cost $s$ in order to learn the price charged by any particular firm as well as her match value for the product sold by that firm. We assume that search cost $s$ is relatively small so that the first search is always worth, that is:

$$0 \leq s \leq \overline{s} \equiv (1 - F(p^m)) \left( \frac{\int_{p^m}^1 \varepsilon f(\varepsilon) d\varepsilon}{1 - F(p^m)} - p^m \right)$$

Moreover, consumers search sequentially with costless recall.

To model the idea that a great deal of the advertising we observe is meant to raise attention in the market, we assume that a firm engages in an advertising race with other firms to lure consumers to visit its shop before they visit the shops of the rivals. In particular, we assume that, at any moment during the search process, a consumer is more likely to go

[5]Janssen and Non (2008) present a related study where some consumers can search at no cost and firms use an all-or-nothing advertising technology.

to a firm $i$ if she has been relatively more exposed to the ads of that firm (or if the ads of firm $i$ have happened to be relatively more salient than other firms' ads).[6]

Let $a_i$, $i = 1, 2, ..., n$ denote the number of advertisements (advertising effort) of firm $i$. We assume that the cost of producing (acquiring) $a_i$ advertisements is $\alpha a_i$. The parameter $\alpha$ represents various advertising technologies. Given an advertising strategy profile $(a_1, a_2, ..., a_n)$, we assume that the probability that a consumer will recall firm $i$ at any moment during the search process is

$$\frac{a_i}{a_i + \sum_{j \neq i} a_j}$$

Therefore, the probability that a consumer will visit firm $i$ in her $k^{\text{th}}$ search, is given by

$$\frac{a_i}{a_i + \sum_{j \neq i} a_j} \prod_{\ell=1}^{k-1} \left(1 - \frac{a_i}{a_i + \sum_{j \neq i} a_j}\right), \; k = 1, 2, ..., n.$$

Our modelling of the consumer recall process is similar to that in the rent-seeking contest described by Tullock (1980).[7] Intuitively, one can think of each advertisement a firm produces as a ball that this firm puts in an urn. Each firm can put as many balls in the urn as it likes. When the consumer happens to need a product and has to start searching for it, it is as if she takes one ball from the urn and visits the firm that has put this particular ball in the urn. If a consumer has already visited a first shop, to decide which of the remaining firms to visit next, she proceeds in the same way: draws again from the urn and visits the firm that has put this particular ball in the urn; and so on and so forth.[8]

The timing in our model is as follows. First, firms simultaneously decide on their advertising investments and prices. Second, consumers sequentially search for the best deal following the recall process described above. We will focus on symmetric pure-strategy equilibria, i.e., where $a_i = a^*$ and $p_i = p^*$, $i = 1, 2, ..., n$. In Section 4 we shall study a market where firms own different advertising technologies.

[6]This is related to the idea in the marketing and business literatures of "top-of-mind awareness" (see e.g. Kotler, 2000). Strictly speaking, our model primarily applies to a set-up where shops attract consumers via advertising messages. Shops cannot advertise the price of their products. There are many instances in which advertising does not convey price information. For example, when shops sell many products (like clothing, electronics, supermarkets, etc.) it is often impractical to list the prices of all products. An alternative interpretation is that consumers simply cannot remember all the prices they have seen in advertisements. The best an advertiser can hope for is that consumers remember its identity.

[7]Schmalensee (1976) uses a similar idea in the context of advertising, but in his model prices are exogenously given. Hortaçsu and Syverson (2004), in their empirical study of price dispersion in the mutual fund industry, also model the funds' sampling probabilities in a similar way. Chioveanu (forthcoming) also uses this type of advertising technology in the context of persuasive (loyalty-inducing) advertising in a market for homogeneous products.

[8]An alternative formulation would be one where firms engage in an advertising race for consumers' attention (akin to the patent race models in the R&D literature). The results in Baye and Hoppe (2003) show that these two formulations are strategically equivalent.

# 3 Analysis

## 3.1 Symmetric firms

Let $(a_{-i}, p_{-i})$ denote the strategies of firms other than firm $i$. Assume that $(a_{-i}, p_{-i}) = (a^*, p^*)$ and consider the (expected) payoff to firm $i$ charging a price $p_i$ and advertising with intensity $a_i$. To calculate firm $i$'s payoff, we need to take into consideration the order in which the firm may be visited and the probability to make a sale conditional on being visited at a given point in time. Assume $p_i \geq p^*$ without loss of generality.

We start by computing the joint probability that consumers visit firm $i$ first and decide to accept the offer of firm $i$ right away without searching further. Suppose that a buyer has approached firm $i$ in her first search and her current purchase option yields utility $\varepsilon_i - p_i$. If $\varepsilon_i - p_i < 0$, the consumer will search again given our assumption $s < \overline{s}$. Suppose $\varepsilon_i - p_i \geq 0$. In the Nash equilibrium, a visit to a new firm $j$ will yield utility $\varepsilon_j - p^*$. Searching one more time yields gains only if the consumer prefers option $j$ over option $i$, i.e., if

$$\varepsilon_j > \varepsilon_i - \Delta \equiv x,$$

with $\Delta = p_i - p^* \geq 0$. Therefore, the expected benefit from searching once more is

$$g(x) \equiv \int_x^1 (\varepsilon - x) f(\varepsilon) d\varepsilon.$$

Searching one more time is worthwhile if and only if these incremental benefits exceed the cost of search $s$. The buyer is exactly indifferent between searching once more and stopping and accepting the offer at hand if $x \geq \hat{x}$, with $\hat{x}$ implicitly defined by

$$g(\hat{x}) = s. \tag{1}$$

The function $g$ is monotonically decreasing. Moreover, $g(0) = \int_0^1 \varepsilon f(\varepsilon) d\varepsilon$ and $g(1) = 0$. It is readily seen that $\overline{s} < \int_0^1 \varepsilon f(\varepsilon) d\varepsilon$. Therefore, for any $s \in [0, \overline{s}]$, there exists a unique $\hat{x} \in [p^m, 1]$ that solves equation (1).

Since in any equilibrium $\hat{x} \geq p^*$, the probability that a buyer stops searching at firm $i$ given that firm $i$ is sampled, is equal to

$$\Pr[x > \hat{x} \text{ and } \varepsilon_i > p_i] = \Pr[x > \hat{x}] = (1 - F(\hat{x} + \Delta)).$$

If we denote the probability that a consumer visits firm $i$ in her first search and buys there right away as $\lambda_1^i(a_i, p_i; a^*, p^*)$ we have:

$$\lambda_1^i(a_i, p_i; a^*, p^*) = \frac{a_i}{a_i + (n-1)a^*}(1 - F(\hat{x} + \Delta)) \tag{2}$$

We now compute the joint probability that a consumer goes to firm $i$ in her second search and decides to acquire the offering of firm $i$ right away. For this we need to take into account that the consumer has visited some other firm first, say firm $j$, but walked away from that firm because the offering was not satisfactory enough. Note that when a consumer walks away from such a firm, the consumer expects to encounter a price equal to $p^*$ in the next shop. Then this probability is given by

$$\Pr[\varepsilon_j < \hat{x} \text{ and } x > \hat{x} \text{ and } \varepsilon_i > p_i] = F(\hat{x})(1 - F(\hat{x} + \Delta)).$$

If we denote by $\lambda_2^i(a_i, p_i; a^*, p^*)$ the chance that firm $i$ is visited in second place and sells its product to the consumer right away we have:

$$\lambda_2^i(a_i, p_i; a^*, p^*) = \left(1 - \frac{a_i}{a_i + (n-1)a^*}\right) \frac{a_i}{a_i + (n-2)a^*} F(\hat{x})(1 - F(\hat{x} + \Delta)) \tag{3}$$

More generally, the joint probability that a consumer visits firm $i$ in her $k^{\text{th}}$, $k = 3, \ldots, n$ search and buys there right away is

$$\lambda_k^i(a_i, p_i; a^*, p^*) = \frac{a_i}{a_i + (n-k)\, a^*} \prod_{\ell=1}^{k-1} \frac{(n-\ell)\, a^*}{a_i + (n-\ell)\, a^*} F(\hat{x})^{k-1} \left[1 - F(\hat{x} + \Delta)\right]. \tag{4}$$

To complete firm $i$'s payoff calculation, we need to compute the joint probability that a consumer walks away from every single firm in the market and happens to return to firm $i$ to conduct a transaction, that is

$$\Pr[\max\{x, \max_{j \neq i}\{\varepsilon_j\}\} < \hat{x} \text{ and } \varepsilon_i - p_i > \max_{j \neq i}\{\varepsilon_j - p^*\} \text{ and } \varepsilon_i > p_i]$$

This probability is independent of the order in which firms are visited so we will denote it as $R(p_i; p^*)$. We then have:

$$R(p_i; p^*) = \int_{p_i}^{\hat{x}+\Delta} F(\varepsilon - \Delta)^{n-1} f(\varepsilon) d\varepsilon. \tag{5}$$

Using the notation introduced above, we can now write firm $i$'s expected profits:

$$\Pi_i(a_i, p_i; a^*, p^*) = p_i \left[\sum_{k=1}^{n} \lambda_k^i(a_i, p_i; a^*, p^*) + R(p_i; p^*)\right] - \alpha a_i. \tag{6}$$

We look for a Nash equilibrium in prices and advertising levels. Thus, we need a price $p^*$ and an advertising level $a^*$ that solve the following first-order conditions:

$$\frac{\partial \Pi_i(a^*, p^*)}{\partial a_i} = p^* \sum_{k=1}^{n} \frac{\partial \lambda_k^i(a^*, p^*)}{\partial a_i} - \alpha = 0, \tag{7}$$

$$\frac{\partial \Pi_i(a^*, p^*)}{\partial p_i} = \sum_{k=1}^{n} \lambda_k^i(a^*, p^*) + R(p^*) + p^* \left[\sum_{k=1}^{n} \frac{\partial \lambda_k^i(a^*, p^*)}{\partial p_i} + \frac{\partial R(p^*)}{\partial p_i}\right] = 0. \tag{8}$$

**Proposition 1** *For every search cost $s \in [0, \overline{s}]$, assume that $\int_{\hat{x}}^{1} \varepsilon f(\varepsilon) d\varepsilon - \hat{x}^2 f(\hat{x}) < s$, where $\hat{x}$ solves (1). Then equilibrium advertising levels are given by*[9]

$$a^* = \frac{p^*}{\alpha n} \left( 1 - F(\hat{x})^n - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k \left(1 - F(\hat{x})^{n-k}\right)}{n-k} \right) \tag{9}$$

*and the equilibrium prices solve*

$$\frac{1 - F(p^*)^n}{n} + p^* \left( -\frac{f(\hat{x})}{n} \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} + \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon \right) = 0 \tag{10}$$

## 3.2 Prices, advertising, profits and the costs of search

In the previous section, we have derived the symmetric equilibrium of our model. In this section, we look at the comparative statics effects of search and advertising costs on prices, advertising intensity and profits.

**Proposition 2** *If $(1 - F)$ is log-concave, then equilibrium prices and advertising intensities are increasing in search cost $s$. Moreover, advertising expenditures fall while prices remain constant as advertising cost $\alpha$ rises.*

Since all the firms advertise with the same intensity in symmetric equilibrium, changes in advertising costs have no price effects and, naturally, greater costs of advertising result in lower advertising levels. The result on the relationship between prices and search costs is similar to Anderson and Renault (1999), who studied a setting where the market is always fully covered in equilibrium. Our Proposition 2 naturally extends their result to a setting where industry demand is elastic. As search costs increase, the probability that a consumer who happens to venture a firm walks away to search for another product falls. This confers market power to the firm and prices therefore increase. The result on the relationship between search costs and advertising costs is novel. Given that an increase in search costs confers a firm more market power over the consumers that it attracts, it becomes more profitable for a firm to lure consumers into its shop. As a result, firms advertise with greater intensity as search costs rise.

The effect of increasing search costs on firm profits is potentially ambiguous because the gains associated to higher prices maybe more than offset by the greater investments in advertising. Our next result shows that the net effect depends on the initial market conditions and the costs of advertising.

**Proposition 3** *For any advertising cost parameter $\alpha$: (A) There exists a sufficiently small search cost $\hat{s}$ such that, for any $s \leq \hat{s}$, profits increase as search cost goes up. (B) Let*

[9] *INCOMPLETE: This condition is necessary for existence of symmetric equilibrium. We still need to rule out "large" deviations.*

*$f(\varepsilon) = 1$ and $n = 2$; then there exists a sufficiently large search cost $\widetilde{s}$ such that, for any $s \geq \widetilde{s}$, profits decrease as search cost rises and eventually profits fall below the profits firms would make in a frictionless world.*

We now elaborate on the intuition behind this result. An increase in search costs has two opposite effects on firm profits. With an increase in $s$, firms gain market power, which allows them to charge a higher price. This effect tends to raise the profits of the firms. At the same time, however, with an increase in $s$ it becomes more attractive for a single firm to invest in saliency and try to beat its rivals in the race for attention. As a result, firms dedicate more resources to advertise as search costs increase, which tends to lower firm profits. Advertising is a rent-seeking activity which leads to a dissipation of the rents generated by greater market power. When search costs are small, the price effect has a dominating influence and firms gain from an increase in search costs. When search costs are large, the negative advertising effect dominates the price effect and profits decrease with higher search costs. Interestingly, the dissipation of rents effect of advertising eventually becomes so severe that firms end up obtaining profits that are lower than the profits they would obtain in a world of perfect information (zero search costs). These observations can be seen in Figure where we plot equilibrium profits against search costs. The dashed lines show the profits firms would make if advertising was banned totally ($a = 0$), as well as the profits firms would make if consumers could search for products costlessly ($s = 0$).

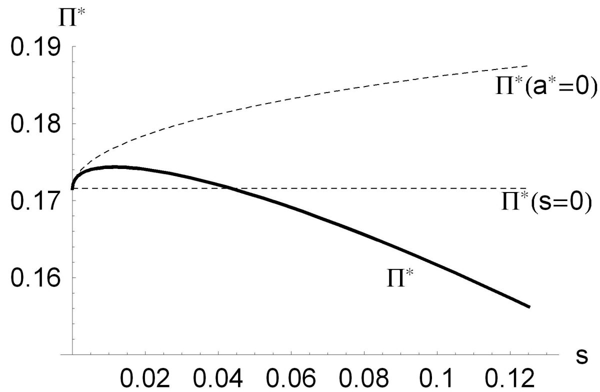


Figure 1: Equilibrium profits ($n = 2$)

In our model, lowering search costs always increases welfare. If the market were fully covered, total welfare would be maximized if investments in advertising were minimized. From Proposition 3, we know this to be the case if search costs are zero. Since we consider a case in which industry demand is not completely inelastic, this result is only reinforced, as lower search costs imply lower prices and hence a lower deadweight loss.

Interestingly, when search costs are sufficiently high, it would even be a Pareto improvement to have lower search costs. Consumers are better off as equilibrium prices decrease, while firms are better off as equilibrium profits increase.[10]

# 4 Asymmetric firms

The analysis in the previous section had all firms using the same advertising technology and therefore exerting the same advertising effort. This implied that in equilibrium all firms were equally likely to be visited by the consumers in a particular place, say the $k^{\text{th}}$ place. Of course, out of equilibrium firms gained by being visited earlier than their rivals and this gave firms incentives to advertise. In this section we explore the implications of different advertising technologies. We are interested in the relationship between advertising budgets and equilibrium prices, profits and welfare.

The simplest setting in which we can ask this question is one where there are just two firms, one with a more efficient advertising technology. Let us then set $N = 2$ in the model above and assume that the advertising costs of one of the firms, say firm 1, are given by $\alpha a_1$, with $\alpha \in [0, 1]$ while the advertising costs of the other firm are simply $a_2$.[11] As we will see later, introducing asymmetry in this model comes at the cost of increasing complexity since we need to solve a non-linear system of four equations for equilibrium. To keep the analysis the most tractable possible, in what follows, we shall assume match values are uniformly distributed.

Let $(a_1^*, p_1^*)$ and $(a_2^*, p_2^*)$ denote the equilibrium strategy profile of the firms. We assume consumers know the equilibrium prices but they do not know which firm charges which price. Let us assume without loss of generality that $p_1^* \geq p_2^*$; we shall ask which advertising efforts are consistent with this being part of equilibrium. Next we compute the profits of the firms. Let us start with firm 1's profits. Suppose that a buyer has approached firm 1 in her first search. Suppose her current purchase option yields utility $\varepsilon_1 - p_1 \geq 0$, where $p_1 \neq p_1^*$ to allow for deviations from equilibrium pricing. In the Nash equilibrium, a visit to firm 2 will yield utility $\varepsilon_2 - p_2^*$. A new search will give the consumer a gain whenever $\varepsilon_2 > \varepsilon_1 - (p_1 - p_2^*) \equiv x_1$. Therefore, the expected benefit from searching once more is $\int_{x_1}^1 (\varepsilon - x_1) f(\varepsilon) d\varepsilon$. Recall that $\hat{x}$ is the solution to $\int_{\hat{x}}^1 (\varepsilon - \hat{x}) f(\varepsilon) d\varepsilon = s$. As argued above, it must be the case that $\hat{x} > p_2^*$ so the probability that a buyer stops at firm 1 given that firm 1 is sampled is equal to $\Pr[x_1 > \hat{x}] = 1 - F(\hat{x} + p_1 - p_2^*)$. Alternatively, the consumer may find it worthwhile to give it a try at firm 2. If the deal at firm 2 is not good enough compared to the one at firm 1, the consumer will return to firm 1 to make a purchase. This occurs with probability

$$\Pr[x_1 < \hat{x} \text{ and } \varepsilon_1 - p_1 > \varepsilon_2 - p_2^* \text{ and } \varepsilon_1 > p_1].$$

[10] Of course, here we are not taking into consideration the advertising industry. If we did, advertisers would lose as search costs fall (advertising expenditures are just transfers from the to the product market to the advertising industry).

[11] We can interpret this assumption as saying that firm 1's advertising is intrinsically more salient than firm 2's, that is, to make a given impact in the market firm 1 needs to spend less resources in advertising than firm 2.

In sum, given the advertising efforts of the firms, the probability that a consumer visits firm 1 in her first search and buys there (either directly or after walking away to visit firm 2 and then come back) is therefore:

$$\frac{a_1}{a_1 + a_2^*}\left(1 - F(\hat{x} + p_1 - p_2^*) + \int_{p_1}^{\hat{x}+p_1-p_2^*} F(\varepsilon_1 - p_1 + p_2^*)dF(\varepsilon_1)\right). \tag{11}$$

Now suppose the consumer visits firm 2 first and observes a deal giving her utility $\varepsilon_2 - p_2^*$. A consumer who goes on with her search and visits firm 1 expects to see a price equal to $p_1^*$. As above, we can define $x_2 \equiv \varepsilon_2 - p_2^* + p_1^*$. Then, the probability a consumer walks away from firm 2 is $\Pr[x_2 < \hat{x}]$. Conditional on visiting firm 2 first, the consumer walks away from firm 2 to visit firm 1, and buys at firm 1 with probability

$$\Pr[x_2 < \hat{x} \text{ and } \varepsilon_1 - p_1 > \varepsilon_2 - p_2^* \text{ and } \varepsilon_1 > p_1].$$

Given the advertising efforts of the firms, the joint probability a consumer visits firm 2 first but buys at firm 1 is:

$$\frac{a_2^*}{a_1 + a_2^*}\left(F(\hat{x} + p_2^* - p_1^*)(1 - F(\hat{x} + p_1 - p_1^*) + \int_{p_1}^{\hat{x}+p_1-p_1^*} F(\varepsilon_1 - p_1 + p_2^*)dF(\varepsilon_1)\right)$$

Then the total profits of firm 1 equal

$$\begin{aligned}\pi_1 &= p_1\frac{a_1}{a_1 + a_2^*}\left(1 - F(\hat{x} + p_1 - p_2^*) + \int_{p_1}^{\hat{x}+p_1-p_2^*} F(\varepsilon_1 - p_1 + p_2^*)dF(\varepsilon_1)\right) \\ &\quad + p_1\frac{a_2^*}{a_1 + a_2^*}\left(F(\hat{x} + p_2^* - p_1^*)(1 - F(\hat{x} + p_1 - p_1^*) + \int_{p_1}^{\hat{x}+p_1-p_1^*} F(\varepsilon_1 - p_1 + p_2^*)dF(\varepsilon_1)\right) - \alpha a_1\end{aligned}$$

With matching values uniformly distributed on $[0, 1]$ we have:

$$\begin{aligned}\pi_1 &= p_1\frac{a_1}{a_1 + a_2^*}\left(1 - \hat{x} - p_1 + p_2^* + \frac{1}{2}(\hat{x}^2 - p_2^{*2})\right) \\ &\quad + p_1\frac{a_2^*}{a_1 + a_2^*}\left((\hat{x} + p_2^* - p_1^*)(1 - \hat{x} - p_1 + p_1^*) + \frac{1}{2}(\hat{x} - p_1^*)(\hat{x} + 2p_2^* - p_1^*)\right) - \alpha a_1\end{aligned}$$

Taking the first order conditions with respect to own advertising intensity and price, and imposing the condition $p_1 = p_1^*$ and $a_1 = a_1^*$ we have, respectively:

$$\begin{aligned}0 &= p_1^*\frac{a_2^*}{(a_1^* + a_2^*)^2}\left(1 - \hat{x} - p_1^* + p_2^* + \frac{1}{2}(\hat{x}^2 - p_2^{*2})\right) \\ &\quad - p_1^*\frac{a_2^*}{(a_1^* + a_2^*)^2}\left((\hat{x} + p_2^* - p_1^*)(1 - \hat{x}) + \frac{1}{2}(\hat{x} - p_1^*)(\hat{x} + 2p_2^* - p_1^*)\right) - \alpha \end{aligned} \tag{12}$$

$$
\begin{aligned}
0 &= \frac{a_1^*}{a_1^*+a_2^*}\left(1-\hat{x}-2p_1^*+p_2^*+\frac{1}{2}(\hat{x}^2-p_2^{*2})\right)\\
&\quad+\frac{a_2^*}{a_1^*+a_2^*}\left((\hat{x}+p_2^*-p_1^*)(1-\hat{x}-p_1^*)+\frac{1}{2}(\hat{x}-p_1^*)(\hat{x}+2p_2^*-p_1^*)\right) \qquad (13)
\end{aligned}
$$

Consider now the problem of firm 2. Let $p_2$ be the price of firm 2 (different than $p_2^*$ to allow for deviations). Suppose consumers visit firm 2 first. Since $\hat{x} > p_1^*$, the probability that a buyer buys directly upon visiting firm 2 is equal to $\Pr[\varepsilon_2 > \hat{x}-(p_1^*-p_2)] = 1-F(\hat{x}+p_2-p_1^*)$. Alternatively, the consumer may want to try the product at firm 1. If the deal at firm 1 is not good enough compared to the one at firm 2, the consumer will return to firm 2 to make a purchase. This occurs with probability

$$\Pr[\varepsilon_2 < \hat{x}-(p_1^*-p_2) \text{ and } \varepsilon_2-p_2 > \varepsilon_1-p_1^* \text{ and } \varepsilon_2 > p_2].$$

In sum, given the advertising efforts of the firms, the probability that a consumer visits firm 2 in her first search and buys there (either directly or after walking away to visit firm 1 and then returning to firm 2) is therefore:

$$\frac{a_2}{a_1^*+a_2}\left(1-F(\hat{x}+p_2-p_1^*)+\int_{p_2}^{\hat{x}+p_2-p_1^*}F(\varepsilon_2-p_2+p_1^*)dF(\varepsilon_2)\right).$$

Suppose now consumers visit firm 1 first so they observe a deal $\varepsilon_1 - p_1^*$. If they walk away from firm 1 they expect to see a price equal to $p_2^*$ at firm 2. Therefore, they will walk away from firm 1 and buy at firm 2 with probability:

$$\Pr[\varepsilon_1 < \hat{x}+p_1^*-p_2^* \text{ and } \varepsilon_2-p_2 > \varepsilon_1-p_1^* \text{ and } \varepsilon_2 > p_2]$$

Taking into account the advertising efforts of the firms, we have that the joint probability consumers visit firm 1 first but end up buying at firm 2 is:

$$\frac{a_1^*}{a_1^*+a_2}\left(F(\hat{x}+p_1^*-p_2^*)(1-F(\hat{x}+p_2-p_2^*)+\int_{p_2}^{\hat{x}+p_2-p_2^*}F(\varepsilon_2-p_2+p_1^*)dF(\varepsilon_2)\right)$$

This shows that the problem of the rival firm is symmetric. So the first order conditions with respect to advertising intensity and price are, respectively:

$$
\begin{aligned}
0 &= p_2^*\frac{a_1^*}{(a_1^*+a_2^*)^2}\left(1-\hat{x}-p_2^*+p_1^*+\frac{1}{2}(\hat{x}^2-p_1^{*2})\right)\\
&\quad-p_2^*\frac{a_1^*}{(a_1^*+a_2^*)^2}\left((\hat{x}+p_1^*-p_2^*)(1-\hat{x})+\frac{1}{2}(\hat{x}-p_2^*)(\hat{x}+2p_1^*-p_2^*)\right)-1 \qquad (14)
\end{aligned}
$$

$$0 = \frac{a_2^*}{a_1^* + a_2^*}\left(1 - \hat{x} - 2p_2^* + p_1^* + \frac{1}{2}(\hat{x}^2 - p_1^{*2})\right)$$
$$+\frac{a_1^*}{a_1^* + a_2^*}\left((\hat{x} + p_1^* - p_2^*)(1 - \hat{x} - p_2^*) + \frac{1}{2}(\hat{x} - p_2^*)(\hat{x} + 2p_1^* - p_2^*)\right) \quad (15)$$

The first order conditions (12),(13),(14) and (15) can be solved numerically to obtain equilibrium advertising levels and prices for different search costs and advertising technologies. Table 1 shows that $p_1^* \geq p_2^*$ is only consistent with advertising levels satisfying $a_1^* < a_2^*$. Therefore, we conclude that the firm with a more efficient advertising technology advertises more and charges a lower price than the rival firm. The reason for the price result is that the firm that advertises more is visited first with higher probability and this makes this firm demand more elastic than the rival firm. This result is in line with the recent study of Armstrong, Vickers and Zhou (2007) on prominence.[12]

We can plug equilibrium advertising efforts and prices in the profits functions to compute equilibrium profits. We note that that the firm that advertises more also obtains a larger profit than the rival firm.

| $s$ | $\hat{x}$ | $\alpha$ | $a_1^*$ | $p_1^*$ | $\Pi_1^*$ | $a_2^*$ | $p_2^*$ | $\Pi_2^*$ |
|---|---|---|---|---|---|---|---|---|
| 0.08 | 0.6 | 1 | 0.01922 | 0.4806 | 0.16557 | 0.01922 | 0.4806 | 0.16557 |
| | | 0.9 | 0.02128 | 0.4802 | 0.16779 | 0.01919 | 0.4811 | 0.16346 |
| | | 0.8 | 0.02370 | 0.4797 | 0.17037 | 0.01904 | 0.4816 | 0.16123 |
| | | 0.7 | 0.02656 | 0.4792 | 0.17340 | 0.01871 | 0.4823 | 0.15889 |
| | | 0.6 | 0.02999 | 0.4787 | 0.17701 | 0.01815 | 0.4831 | 0.15644 |
| | | 0.5 | 0.03414 | 0.4782 | 0.18135 | 0.01728 | 0.4840 | 0.15389 |
| | | 0.4 | 0.03925 | 0.4777 | 0.18665 | 0.01595 | 0.4852 | 0.15128 |
| | | 0.3 | 0.04565 | 0.4772 | 0.19322 | 0.01397 | 0.4868 | 0.14867 |
| | | 0.2 | 0.05383 | 0.4768 | 0.20155 | 0.01104 | 0.4887 | 0.14617 |
| | | 0.1 | 0.06457 | 0.4764 | 0.21237 | 0.00666 | 0.4914 | 0.14400 |
| | | 0 | 0.07916 | 0.4763 | 0.22687 | 0 | 0.4953 | 0.14253 |
| 0.02 | 0.8 | 1 | 0.0044 | 0.4454 | 0.17406 | 0.0045 | 0.4454 | 0.17406 |
| | | 0.9 | 0.0049 | 0.4447 | 0.17472 | 0.0044 | 0.4460 | 0.17343 |
| | | 0.8 | 0.0055 | 0.4441 | 0.17548 | 0.0044 | 0.4467 | 0.17276 |
| | | 0.7 | 0.0061 | 0.4434 | 0.17637 | 0.0043 | 0.4476 | 0.17204 |
| | | 0.6 | 0.0069 | 0.4427 | 0.17743 | 0.0042 | 0.4486 | 0.17129 |
| | | 0.5 | 0.0079 | 0.4419 | 0.17869 | 0.0040 | 0.4499 | 0.17049 |
| | | 0.4 | 0.0091 | 0.4411 | 0.18023 | 0.0037 | 0.4514 | 0.16966 |
| | | 0.3 | 0.0106 | 0.4403 | 0.18213 | 0.0033 | 0.4533 | 0.16879 |
| | | 0.2 | 0.0125 | 0.4395 | 0.18454 | 0.0026 | 0.4558 | 0.16792 |
| | | 0.1 | 0.0150 | 0.4387 | 0.18766 | 0.0016 | 0.4590 | 0.16709 |
| | | 0 | 0.0184 | 0.4380 | 0.19187 | 0 | 0.4634 | 0.16636 |

Table 1: Model with asymmetric firms.
Equilibrium advertising efforts, prices and profits
(low (s=0.02) and high (s=0.08) search costs).

We now present the welfare calculations. To compute consumer surplus, we need to

[12]In Armstrong et al. (2007) firms cannot influence the order in which they are visited. Their results correspond to the case in our paper in which advertising levels are set exogenously to $a_i^* > 0$ and $a_j^* = 0$, $i, j = 1, 2$, $i \neq j$, so that one of the firms is visited first with probability 1.

take into consideration the various situations a consumer may encounter herself. First, a consumer may recall first either firm 1 or firm 2. Suppose a consumer recalls first firm 1. Suppose also that the consumer finds and acceptable match at firm 1 and therefore buys the product right away. Let $CS(1 \mid 1)$ denote the expected consumer surplus in this case (buying at firm 1 right after visiting firm 1). Then we have:

$$CS(1 \mid 1) = \int_0^1 \int_{\hat{x}-p_2^*+p_1^*}^1 (\varepsilon_1 - p_1^* - s) d\varepsilon_i d\varepsilon_j$$

When $\alpha$ gets smaller, $p_1^*$ decreases and $p_2^*$ increases. Then it is clear that with lower advertising costs more consumers buy right away upon arriving at firm 1 and each of these consumers obtains a greater surplus.

Some consumers visit firm 1 first, then they go to firm 2 and finally end up coming back to firm 1 to buy there. Using the notation $CS(1 \mid 1, 2)$, the expected surplus of a consumer is:

$$CS(1 \mid 1, 2) = \int_{p_1^*}^{\hat{x}-p_2^*+p_1^*} \int_0^{\varepsilon_1+p_2^*-p_1^*} (\varepsilon_1 - p_1^* - 2s) d\varepsilon_1 d\varepsilon_2$$

The number of consumers in this group is $(\hat{x}^2 - p_2^{*2})/2$, and since $p_2^*$ increases when $\alpha$ falls, fewer consumers come back to buy from firm 1 after visiting first firm 1 and then firm 2. Since these consumers derive a greater surplus, the total effect is ambiguous.

Finally, the consumer may fail to find a satisfactory product at firm 1, walk away from firm 1 to visit firm 2 and buy at firm 2. Denoting the consumer surplus in this case as $CS(2 \mid 1, 2)$ (buying at firm 2 after visiting 1 and 2), in this case consumer surplus is:

$$CS(2 \mid 1, 2) = \int_{\hat{x}}^1 \int_0^{\hat{x}-p_2^*+p_1^*} (\varepsilon_2 - p_2^* - 2s) d\varepsilon_1 d\varepsilon_2 + \int_{p_2^*}^{\hat{x}} \int_0^{\varepsilon_2-p_2^*+p_1^*} (\varepsilon_2 - p_2^* - 2s) d\varepsilon_1 d\varepsilon_2$$

It is straightforward to see that these consumers fall in number, and since each of them pays a higher price when $\alpha$ falls, it is clear that the surplus they derive decreases.

Similar considerations apply when the consumer recalls firm 2 first. Using the expressions above, ex-ante expected consumer surplus is:

$$\begin{aligned} CS &= \frac{a_1^*}{a_1^* + a_2} (CS(1 \mid 1) + CS(2 \mid 1, 2) + CS(1 \mid 1, 2)) \\ &\quad + \frac{a_2^*}{a_1^* + a_2} (CS(2 \mid 2) + CS(1 \mid 1, 2) + CS(2 \mid 1, 2)) \end{aligned} \tag{16}$$

and welfare is $W = CS + \Pi_1^* + \Pi_2^*$ as usual.

In sum, as firm 1's advertising cost parameter $\alpha$ decreases, the price of firm 1 goes down while the price of firm 2 increases. The first effect is good for consumers because they happen to visit the cheapest firm in first place more frequently than the more expensive one; however, since firm 2's price is higher, consumers who fail to find a satisfactory product at firm 1 are forced to accept it anyway more often. Consumers on average will search less, which on the one hand is good because of lower search costs but on the other hand it makes consumers less exposed to variety. The aggregate implications of greater efficiency in firm 1's advertising on consumer surplus are therefore complex. Some consumers gain while others lose. An important fact is that the total number of consumers who participate in the market, which is given by $1 - p_1^* p_2^*$ decreases as firm 1 becomes more advertising-efficient. This is an important source of inefficiency. In addition, we note that advertising outlays decrease, which has a positive effect on welfare.

The total effects of decreasing firm 1's advertising costs are reported in Table 2. We note that aggregate firm profits increase, aggregate consumer surplus decreases and welfare increases as the advertising costs of firm 1 go down. Consumer surplus decreases so the welfare result is due to lower advertising costs. In fact, it turns out that welfare gross of advertising costs also decreases as $\alpha$ decreases. We see that, in spite of firm 1's lower price, the increase in the price of firm 2 has a dominating influence thus generating a larger deadweight loss.

| $s$ | $\hat{x}$ | $\alpha$ | $\sum \Pi^*$ | $CS$ | $W$ | Search costs | Advertising |
|---|---|---|---|---|---|---|---|
| 0.08 | 0.6 | 1 | 0.33115 | 0.121343 | 0.452490 | 0.09104 | 0.03845 |
| | | 0.9 | 0.33125 | 0.121340 | 0.452591 | 0.09103 | 0.03834 |
| | | 0.8 | 0.33160 | 0.121326 | 0.452929 | 0.09101 | 0.03799 |
| | | 0.7 | 0.33229 | 0.121301 | 0.453593 | 0.09098 | 0.03730 |
| | | 0.6 | 0.33344 | 0.121257 | 0.454702 | 0.09091 | 0.03615 |
| | | 0.5 | 0.33524 | 0.121190 | 0.456427 | 0.09081 | 0.03435 |
| | | 0.4 | 0.33793 | 0.121088 | 0.459014 | 0.09066 | 0.03165 |
| | | 0.3 | 0.34189 | 0.120935 | 0.462829 | 0.09043 | 0.02766 |
| | | 0.2 | 0.34773 | 0.120707 | 0.468435 | 0.09008 | 0.02180 |
| | | 0.1 | 0.35637 | 0.120361 | 0.476729 | 0.08956 | 0.01312 |
| | | 0 | 0.36941 | 0.119816 | 0.489222 | 0.08874 | 0 |
| 0.02 | 0.8 | 1 | 0.34812 | 0.221350 | 0.569469 | 0.028066 | 0.008907 |
| | | 0.9 | 0.34814 | 0.221342 | 0.569487 | 0.028064 | 0.008884 |
| | | 0.8 | 0.34824 | 0.221313 | 0.569549 | 0.028058 | 0.008803 |
| | | 0.7 | 0.34842 | 0.221256 | 0.569671 | 0.028046 | 0.008645 |
| | | 0.6 | 0.34872 | 0.221160 | 0.569876 | 0.028026 | 0.008380 |
| | | 0.5 | 0.34918 | 0.221010 | 0.570193 | 0.027995 | 0.007968 |
| | | 0.4 | 0.34988 | 0.220783 | 0.570670 | 0.027948 | 0.007348 |
| | | 0.3 | 0.35093 | 0.220446 | 0.571373 | 0.027879 | 0.006431 |
| | | 0.2 | 0.35246 | 0.219943 | 0.572406 | 0.027774 | 0.005078 |
| | | 0.1 | 0.35475 | 0.219184 | 0.573934 | 0.027617 | 0.003063 |
| | | 0 | 0.35823 | 0.218001 | 0.576231 | 0.027372 | 0 |

Table 2: Model with asymmetric firms.
Aggregate profits, consumer surplus, welfare, search and advertising costs
(low (s=0.02) and high (s=0.08) search costs).

# 5 Conclusion

In this paper, we have modelled the idea that, in an attempt to being visited as early as possible in the course of search of a consumer, firms engage themselves in a battle for attention. Through investments in more and more appealing advertising, a firm can achieve a salient place in consumer awareness so that consumers will visit this firm sooner when searching for a product they need. Advertising is not a winner-takes-all contest in our setting: when a consumer does come to a firm first, she can still decide to go to a different firm if she does not like the product of this particular firm, or if she thinks it is too expensive.

We have found that prices and advertising levels are increasing in consumers' search costs. Yet, the effect on profits is ambiguous. If search costs are small to start with, then firms are better off if search costs increase. Instead, when search costs are already high a further increase in search costs lowers firm profits. In the latter case, getting the attention of a consumer becomes so important that firms over-dissipate the rents generated by being visited earlier than rival firms. This highlights the importance of looking at the interaction of advertising and search costs, rather than only looking at search costs or advertising in isolation. We believe this to be a general phenomenon, that applies beyond the scope of this particular model.

Another interesting finding is that firms with more efficient advertising technologies advertise more, charge lower prices and obtain greater profits than less efficient rivals. Moreover, an increase in advertising cost asymmetries leads to a fall in consumer surplus. This is because, even though advertising serves to direct consumers to better deals on average, less advertising-efficient rivals increase their prices so much that ultimately fewer consumers participate in the market in equilibrium. Firms advertising cost asymmetries weaken firm advertising competition and it turns out that cuts in advertising outlays lead to welfare gains.

# 6 Appendix

**Proof of Proposition 1**

Using the expressions (2)-(4), we can compute

$$\frac{\partial \lambda_1^i}{\partial a_i} = \frac{(n-1)\,a^*}{(a_i+(n-1)\,a^*)^2}(1-F(\hat{x}+\Delta))$$

$$\cdots$$

$$\frac{\partial \lambda_k^i}{\partial a_i} = \left[\frac{a_i}{a_i+(n-k)\,a^*}\sum_{\ell=1}^{k-1}\left[\frac{-(n-\ell)\,a^*}{(a_i+(n-\ell)\,a^*)^2}\prod_{m\neq\ell}^{k-1}\frac{(n-m)\,a^*}{a_i+(n-m)\,a^*}\right]\right.$$
$$\left.+\frac{(n-k)\,a^*}{(a_i+(n-k)\,a^*)^2}\prod_{\ell=1}^{k-1}\frac{(n-\ell)\,a^*}{a_i+(n-\ell)\,a^*}\right]F(\hat{x})^{k-1}(1-F(\hat{x}+\Delta))$$

$$\cdots$$

$$\frac{\partial \lambda_n^i}{\partial a_i} = \sum_{\ell=1}^{n-1}\left[\frac{-(n-\ell)\,a^*}{(a+(n-\ell)\,a^*)^2}\prod_{m\neq\ell}^{n-1}\frac{(n-m)\,a^*}{a+(n-m)\,a^*}\right]F(\hat{x})^{n-1}(1-F(\hat{x}+\Delta))$$

In symmetric equilibrium we have

$$\frac{\partial \lambda_1^i}{\partial a_i} = \frac{n-1}{n^2 a^*}\left(1-F(\hat{x})\right)$$

$$\cdots$$

$$\frac{\partial \lambda_k^i}{\partial a_i} = \left[\frac{1}{n-k+1}\sum_{\ell=1}^{k-1}\left[\frac{-(n-\ell)}{(n-\ell+1)^2\,a^*}\prod_{m\neq\ell}^{k-1}\frac{n-m}{n-m+1}\right]\right.$$
$$\left.+\frac{n-i}{(n-i+1)^2\,a^*}\prod_{\ell=1}^{k-1}\frac{n-\ell}{n-\ell+1}\right]F(\hat{x})^{k-1}\left(1-F(\hat{x})\right)$$

$$\cdots$$

$$\frac{\partial \lambda_n^i}{\partial a_i} = \sum_{\ell=1}^{n-1}\left[\frac{-(n-\ell)}{(n-\ell+1)^2\,a^*}\prod_{m\neq\ell}^{n-1}\frac{n-m}{n-m+1}\right]F(\hat{x})^{n-1}\left(1-F(\hat{x})\right).$$

Note that

$$\prod_{\ell=1}^{k-1}\frac{n-\ell}{n+1-\ell} = \frac{n-1}{n}\cdot\frac{n-2}{n-1}\cdot\ldots\cdot\frac{n+1-k}{n+2-k} = \frac{n+1-k}{n}.$$

This allows us to simplify some expressions, in particular:

$$\begin{aligned}
\frac{\partial \lambda_k^i}{\partial a_i} &= \left[\frac{1}{n-k+1}\sum_{\ell=1}^{k-1}\left[\frac{-1}{(n-\ell+1)^2 a^*}\left(\frac{n+1-k}{n}\right)(n-\ell+1)\right]\right. \\
&+ \left.\frac{n-k}{(n-k+1)\,na^*}\right] F(\hat{x})^{k-1}\left(1-F(\hat{x})\right) \\
&= \frac{1}{na^*}\left[\frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1}\frac{1}{(n-\ell+1)}\right] F(\hat{x})^{k-1}\left(1-F(\hat{x})\right), \text{ for all } k=1,2,...,n.
\end{aligned}$$

Moreover

$$\sum_{k=1}^{n}\lambda_k(a^*,p^*) = \frac{1}{n}(1-F(\hat{x})^n)$$

Using these derivations and the expression for $R(p^*)$ in (5) above, the first order conditions in (7) and (8) can be rewritten as:

$$\begin{aligned}
&p^*\sum_{k=1}^{n}\frac{1}{na^*}\left(\frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1}\frac{1}{(n-\ell+1)}\right) F(\hat{x})^{k-1}\left(1-F(\hat{x})\right) - \alpha = 0, \\
&\frac{1-F(\hat{x})^n}{n} + \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f(\varepsilon) d\varepsilon \\
&+p^*\left(-\frac{f(\hat{x})}{n}\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - \int_{p^*}^{\hat{x}}(n-1)F(\varepsilon)^{n-2}f(\varepsilon)^2 d\varepsilon - F(p^*)^{n-1}f(p^*) + F(\hat{x})^{n-1}f(\hat{x})\right) = 0.
\end{aligned}$$

Let us denote

$$C_k \equiv \frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1}\frac{1}{n-\ell+1}.$$

Using the integration by parts rule,[13] these equations can be simplified to:

$$p^*\frac{1}{na^*}\left(1-F(\hat{x})\right)\sum_{k=1}^{n} C_k F(\hat{x})^{k-1} - \alpha = 0, \tag{17}$$

$$\frac{1-F(p^*)^n}{n} + p^*\left(-\frac{f(\hat{x})}{n}\frac{1-F(\hat{x})^n}{1-F(\hat{x})} + \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon\right) = 0. \tag{18}$$

Consider equation (18). To study the existence of a solution in $p^*$, it is useful to rewrite it as follows:

$$\frac{1-F(p^*)^n}{np^*} = \frac{f(\hat{x})}{n}\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon. \tag{19}$$

Note that the LHS of (19) is a positive-valued function that decreases monotonically in $p^*$. Moreover, when $p^* \to 0$ the LHS goes to $\infty$. The RHS, by contrast is monotonically

[13] $\int_a^b u dv = uv|_a^b - \int_a^b v du$.

increasing in $p^*$. Therefore, a solution satisfying $p^* < \hat{x}$ exists if and only if the following condition holds:

$$\frac{1 - F(\hat{x})}{\hat{x} f(\hat{x})} < 1 \tag{20}$$

where $\hat{x}$ follows from (1). Using the definition of $\hat{x}$ above, it is straightforward to verify that the condition in the proposition is equivalent to (20).

Consider now equation (17). Solving for $a^*$ we get:

$$a^* = \frac{p^*}{\alpha n} (1 - F(\hat{x})) \sum_{k=1}^{n} Ck \cdot F(\hat{x})^{k-1},$$

Note that

$$\begin{aligned} C_k - C_{k-1} &= \left( \frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell+1} \right) - \left( \frac{n-k+1}{n-k+2} - \sum_{\ell=1}^{k-2} \frac{1}{n-\ell+1} \right) \\ &= \frac{n-k}{n-k+1} - \frac{1}{n-k+2} - \frac{n-k+1}{n-k+2} = \frac{-1}{n-k+1}. \end{aligned}$$

We thus have

$$C_k = C_{k-1} - \frac{1}{n-k+1},$$

which implies that

$$C_k = \frac{n-1}{n} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell}.$$

For the equilibrium advertising level we then have

$$\begin{aligned} a^* &= \frac{p^*}{\alpha n} (1 - F(\hat{x})) \sum_{k=1}^{n} \left[ \frac{n-1}{n} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell} \right] \cdot F(\hat{x})^{k-1} \\ &= \frac{p^*}{\alpha n} (1 - F(\hat{x})) \left[ \frac{n-1}{n} \sum_{k=1}^{n} F(\hat{x})^{k-1} - \sum_{k=1}^{n} \sum_{\ell=1}^{k-1} \frac{1}{n-\ell} \cdot F(\hat{x})^{k-1} \right] \\ &= \frac{p^*}{\alpha n} (1 - F(\hat{x})) \left[ \frac{n-1}{n} \sum_{k=1}^{n} F(\hat{x})^{k-1} - \sum_{\ell=1}^{n-1} \left( \frac{1}{n-\ell} \sum_{k=\ell+1}^{n} F(\hat{x})^{k-1} \right) \right] \\ &= \frac{p^*}{\alpha n} (1 - F(\hat{x})) \left[ \frac{n-1}{n} \sum_{k=0}^{n-1} F(\hat{x})^{k} - \sum_{\ell=1}^{n-1} \left( \frac{1}{n-\ell} \sum_{k=\ell}^{n-1} F(\hat{x})^{k} \right) \right], \end{aligned}$$

which can be further simplified to

$$\begin{aligned}
a^* &= \frac{p^*}{\alpha n}(1-F(\hat{x}))\left[\frac{n-1}{n}\cdot\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - \sum_{\ell=1}^{n-1}\left(\frac{1}{n-\ell}\right)\frac{F(\hat{x})^\ell - F(\hat{x})^n}{1-F(\hat{x})}\right] \\
&= \frac{p^*}{\alpha n}\left[\frac{n-1}{n}\cdot(1-F(\hat{x})^n) - \sum_{\ell=1}^{n-1}\left(\frac{1}{n-\ell}\right)F(\hat{x})^\ell\left(1-F(\hat{x})^{n-\ell}\right)\right] \\
&= \frac{p^*}{\alpha n}\left[1-F(\hat{x})^n - \sum_{k=0}^{n-1}\frac{F(\hat{x})^k\left(1-F(\hat{x})^{n-k}\right)}{n-k}\right].
\end{aligned}$$

■

**Proof of Proposition 2**.

We build on the proof of Proposition 1. The equilibrium price is given by the solution of the following equation:

$$\frac{1-F(p^*)^n}{np^*} = \frac{f(\hat{x})}{n}\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1}f'(\varepsilon)d\varepsilon. \tag{21}$$

Notice that in this equation the effects of higher search costs are manifested only through changes in $\hat{x}$. Note also that the LHS of (21) decreases in $p^*$ and does not depend on $\hat{x}$. Therefore we only need to study how the RHS of this equation changes with $\hat{x}$. From above, we know that the RHS of (21) is monotonically increasing in $p^*$ so if the RHS increases in $\hat{x}$, then the price decreases as $\hat{x}$ goes up, and, since $\hat{x}$ and search costs $s$ are inversely related, it increases as $s$ goes down.

Taking the derivative of the RHS of (21) with respect to $\hat{x}$ yields:

$$\frac{\left[f'(\hat{x})(1-F(\hat{x})^n) - nF(\hat{x})^{n-1}f^2(\hat{x})\right](1-F(\hat{x})) + f(\hat{x})^2(1-F(\hat{x})^n)}{n(1-F(\hat{x}))^2} - F(\hat{x})^{n-1}f'(\hat{x}) \tag{22}$$

At this point we can follow the steps in Anderson and Renault (1999), which shows that their proof extends naturally to a situation where the market is not covered. For completeness, we provide the last steps. The expression in (22) can be written as:

$$\frac{\left[f'(\hat{x})(1-F(\hat{x})) + f^2(\hat{x})\right](1-F(\hat{x})^n)}{n(1-F(\hat{x}))}\left[\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - nF(\hat{x})^{n-1}\right] \tag{23}$$

The first term is positive because of log concavity of $1-F$. The second term is also positive because it equals $\sum_{k=0}^{n-1}\left[F(\hat{x})^k - F(\hat{x})^{n-1}\right]$ and $F$ is a distribution function.

We now prove that advertising intensities also increase as search costs go up. For simplicity, we rewrite $a^*$ as

$$a^* = \frac{p^*A}{n},$$

with

$$A \equiv (1 - F(\hat{x})^n) - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k \left(1 - F(\hat{x})^{n-k}\right)}{n-k}.$$

We take the derivative of $a^*$ with respect to $\hat{x}$. From (1), we immediately have that $\hat{x}$ is decreasing in $s$. Hence, for $a^*$ to be increasing in $s$, we need it to be decreasing in $\hat{x}$. We have:

$$\frac{\partial a^*}{\partial \hat{x}} = \frac{A}{n}\frac{\partial p^*}{\partial \hat{x}} + \frac{p^*}{n}\frac{\partial A}{\partial \hat{x}} < 0,$$

From the discussion above we know that $\partial p^*/\partial \hat{x} < 0$. Therefore, if we show that $\partial A/\partial \hat{x} < 0$, the results follows. Dropping subscripts, we have

$$\begin{aligned}
\frac{\partial A}{\partial \hat{x}} &= -nF^{n-1}f - F^{n-1}f - \sum_{k=1}^{n-1} \frac{kF^{k-1}\left(1 - F^{n-k}\right) - (n-k)\, F^k F^{n-k-1}}{n-k} f \\
&= -nF^{n-1}f - F^{n-1}f - \sum_{k=1}^{n-1} \frac{kF^{k-1}\left(1 - F^{n-k}\right)}{n-k} f + \sum_{k=0}^{n-1} F^{n-1} f \\
&= -F^{n-1}f - \sum_{k=1}^{n-1} \frac{kF^{k-1}\left(1 - F^{n-k}\right)}{n-k} f < 0.
\end{aligned}$$

The result on the relationship between advertising costs and prices and advertising intensities follows straightforwardly from the first order conditions () and (). ■

**Proof of Proposition 3**

Plugging our expression for $a^*$ into the profit function yields:

$$\begin{aligned}
\Pi_i(a^*, p^*) &= p^* \sum_{k=1}^{n} \frac{1}{n} F(\hat{x})^{k-1} (1 - F(\hat{x})) + p^* \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon \\
&\quad - \frac{p^*}{n} \left( (1 - F(\hat{x})^n) - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k \left(1 - F(\hat{x})^{n-k}\right)}{n-k} \right) \\
&= p^* \left[ \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon + \frac{1}{n} \sum_{k=0}^{n-1} \frac{F(\hat{x})^k \left(1 - F(\hat{x})^{n-k}\right)}{n-k} \right].
\end{aligned}$$

For ease of exposition, we will write equilibrium profits as

$$\Pi_i(\cdot) = p^* T(p^*, \hat{x}),$$

with

$$T(p^*, \hat{x}) \equiv \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon + \frac{1}{n} \sum_{k=0}^{n-1} \frac{F(\hat{x})^k \left(1 - F(\hat{x})^{n-k}\right)}{n-k},$$

and $p^*$ is given by the solution to (6). Again, we take derivatives with respect to reservation utility $\hat{x}$. We have

$$\frac{\partial \Pi(\cdot)}{\partial \hat{x}} = \frac{\partial p^*}{\partial \hat{x}} \left( T(\cdot) + p^* \frac{\partial T(\cdot)}{\partial p^*} \right) + p^* \frac{\partial T(\cdot)}{\partial \hat{x}}.$$

where

$$\frac{\partial p^*}{\partial \hat{x}} = -\frac{\frac{\left[f'(\hat{x})(1-F(\hat{x}))+f^2(\hat{x})\right](1-F(\hat{x})^n)}{n(1-F(\hat{x}))} \left[\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - nF(\hat{x})^{n-1}\right]}{-\frac{nF(p^*)f(p^*)p^*-(1-F(p^*)^n)}{np^{*2}} - F(p^*)^{n-1}f'(p^*)},$$

$$\frac{\partial T(\cdot)}{\partial p^*} = -F(p^*)^{n-1}f(p^*),$$

and

$$\frac{\partial T(\hat{x})}{\partial \hat{x}} = \sum_{k=1}^{n-1} \frac{F(\hat{x})^{k-1}\left(k - nF(\hat{x})^{n-k}\right)}{n-k} f(\hat{x}).$$

To get the last derivative, we have first taken the case $k = 0$ outside of the summation. The derivative of this term then exactly drops out against the derivative of the first term in $T$.

To prove (A), consider the case where search costs are very small: $s \to 0$. Then $\hat{x} \to 1$, so $F(\hat{x}) \to 1$. In this case, from Proposition 1 it is straightforward to verify that $\lim_{s\to 0} p^* > 0$. Moreover since $\lim_{F\to 1}(1 - F^n)/(1 - F) = n$, we have that

$$\lim_{s\to 0} \frac{\partial p^*}{\partial \hat{x}} = 0.$$

Also

$$\lim_{s\to 0} T(\cdot) = 1,$$
$$\lim_{s\to 0} \frac{\partial T(\cdot)}{\partial p^*} < 0,$$
$$\lim_{s\to 0} \frac{\partial T(\cdot)}{\partial \hat{x}} = -(n-1)\, f(1) < 0$$

Taken these terms together, this implies

$$\lim_{s\to 0} \frac{\partial \Pi(\cdot)}{\partial \hat{x}} = -(n-1)\, f(1) \lim_{s\to 0} p^* < 0.$$

Now consider the (B) case in which search costs are high $s \to \overline{s}$, match values are uniformly distributed and two firms operate in the industry. In this case, the equilibrium of the model is given by:

$$p^* = \frac{1}{2}\left(\sqrt{2s} - 2 + \sqrt{8 - 4\sqrt{2s} + 2s}\right),$$
$$a^* = \frac{sp^*}{2\alpha},$$
$$\Pi^* = \frac{1}{2}p^*(1 - p^{*2} - s),$$

where $s$ ranges from 0 to 1/8 in this case. It is straightforward to verify that $\Pi^*$ is a strictly concave function reaching a maximum at $s = 0.0115631$. Moreover, equilibrium profits are always lower than profits in a frictionless world (zero search cost) as long as search cost is sufficiently large. ■

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