# Exploiting Naivete about Self-Control in the Credit Market*

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**Abstract**

We analyze behavior and welfare in a competitive credit market where borrowers with different tastes for immediate gratification ($\beta$) and prior beliefs about that taste ($\hat{\beta}$) sign long-term contracts specifying a loan amount and a menu of repayment schedules. Consistent with many credit-card and mortgage contracts, a competitive-equilibrium contract features an advantageous schedule involving overly front-loaded repayment, but imposes a discontinuous fee for any delay relative to this schedule. Fully sophisticated borrowers ($\hat{\beta} = \beta$) repay at the fast rate, but all other borrowers ($\hat{\beta} > \beta$), including those with an arbitrarily small amount of naivete, put off the bulk of the repayment to later. Furthermore, because non-sophisticated borrowers believe they will repay quickly, they underestimate the cost of credit, and hence borrow too much given their preferences at the time of signing. Due to these mistakes, non-sophisticated consumers have discontinuously lower welfare than sophisticated ones. We identify natural conditions under which the above results obtain even if firms observe neither $\beta$ nor $\hat{\beta}$—because all non-sophisticated consumers endogenously choose contracts for which they discretely mispredict their future behavior. Requiring credit contracts to have a linear structure prevents non-sophisticated but not-too-naive borrowers from severely mispredicting their behavior, so this intervention can raise welfare.

**Keywords:** hyperbolic discounting, sophistication, partial naivete, consumer protection, subprime markets, credit cards

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# 1 Introduction

A growing empirical literature on savings and credit suggests that consumers have a time-inconsistent taste for immediate gratification, and are often naive about this taste.[1] Motivated in part by similar intuitions regarding borrowers' behavior, numerous policymakers, journalists, and academics, have raised concerns that some debt contracts may be harmful to consumers. Bar-Gill (2004) and Warren (2007) warn that credit-card contracts are deliberately loaded with "tricks and traps" to make unwitting consumers stumble and "bleed" consumers after they stumble. Similarly, Engel and McCoy (2002) argue that some features of subprime mortgages are designed to exploit naive consumers, and Fellowes (2006) makes the same observation regarding financial services in low-income neighborhoods more generally.[2] Yet these economic and "consumer-protection" literatures remain largely disconnected: the former one does not consider the welfare effects of credit contracts, while the latter one is not built on a solid theoretical foundation.

In this paper, we provide a formal economic analysis of the welfare effects of credit contracts when some consumers have a time-inconsistent taste for immediate gratification that they may only partially understand. Our theory predicts that to attract consumers while ensuring non-trivial profits from even trivial amounts of naivete, a firm offers seemingly attractive credit to be repaid quickly, but introduces a discontinuous penalty for falling behind this baseline repayment schedule. Consistent with this prediction, most credit cards do not charge interest on *any* purchases if the borrower pays the entire balance due each month, but do charge interest on *all* purchases if

[1] Laibson, Repetto, and Tobacman (1998, 2007) estimate that to explain a typical household's simultaneous holdings of substantial illiquid wealth and credit-card debt, the household's short-term discount rate must be higher than the long-term discount rate. Because many households are calculated to be made worse off by owning credit cards, the fact that they get those cards suggests some degree of naivete about future use. Consistent with this idea, consumers overrespond to the introductory "teaser" rates in credit-card solicitations relative to the length of the introductory period (Shui and Ausubel 2004) and the post-introductory interest rate (Ausubel 1999)—suggesting that they end up borrowing more than they intended or expected. Finally, Skiba and Tobacman (2007) find that the majority of payday borrowers default on a loan, yet do so only after paying significant costs to service their debt. Calibrations indicate that such costly delay in default is only consistent with partially naive time inconsistency. For further discussions and evidence, see Bertaut and Haliassos (2002) and DellaVigna and Malmendier (2004). And bolstering the literature on savings and credit is research by DellaVigna and Paserman (2005) and Paserman (2006) on job search, Oster and Scott Morton (2005) on magazine subscription prices, DellaVigna and Malmendier (2006) on health-club attendance, and Fang and Silverman (2007) on welfare-program participation, which finds evidence of a taste for immediate gratification in these other domains.

[2] As evidenced for instance by Manning's bestseller book *Credit Card Nation* and the documentary movie *Maxed Out*, the question of consumer behavior and welfare in the credit market also receives a lot of popular attention.

she revolves even \$1.[3] In the same vein, about 80% of subprime mortgages feature (typically large) refinancing penalties (Renuart 2004). We show that these discontinuities have some adverse welfare consequences. Most importantly, if a borrower is not *perfectly* sophisticated—if she mispredicts her time inconsistency by even a little bit—she chooses a contract with which she ends up giving in to her taste for immediate gratification and delaying part of the repayment despite the penalty. To make matters worse, because she falsely believes she will not delay, she tends to underestimate the cost of credit and borrow too much given her preferences at the time of borrowing. Hence, a policy of disallowing discontinuous penalties, for example by requiring credit contracts to have a simple linear structure, can raise welfare.

Section 3 presents our model, which adapts theories by DellaVigna and Malmendier (2004), Kőszegi (2005), and Eliaz and Spiegler (2006) to credit markets, and extends these theories by allowing for heterogeneity in the taste for immediate gratification as well as naivete. There are three periods, 0, 1, and 2. From a period-0 perspective, the borrower's utility is $c - k(q) - k(r)$, where $c$ is the amount she borrows and consumes, $q$ and $r$ are the amounts she repays in periods 1 and 2, respectively, and $k(\cdot)$ is a strictly increasing and strictly convex function representing the cost of repayment. Self 1, the period-1 incarnation of the individual, acts to maximize $-k(q) - \beta k(r)$ for some $0 < \beta \leq 1$. For $\beta < 1$, the consumer has a time-inconsistent taste for immediate gratification: in period 1, she puts lower weight on the period-2 cost of repayment than she would have preferred earlier. Consistent with much of the literature, we take the long-term perspective and equate the consumer's welfare with self 0's utility, but the overborrowing we find means that self 1 is also hurt by a non-sophisticated borrower's contract choice. To allow for self 0 to be overoptimistic regarding her future taste for immediate gratification, we assume that she believes she will maximize $-k(q) - \hat{\beta} k(r)$ in period 1, so that $\hat{\beta}$ satisfying $\beta \leq \hat{\beta} \leq 1$ represents her beliefs about $\beta$.

The consumers introduced above contract with competitive suppliers of credit, who have funds available to them at an interest rate of zero. In the bulk of the paper, we consider situations

[3] Although we have not found a systematic analysis of what percentage of credit-card contracts have this feature, it is the majority based on a casual search and is consistently emphasized in online advice for consumers. Type "credit card" and "grace period" in Google or see the following pages (accessed November 2007):
http://home3.americanexpress.com/corp/consumerinfo/grace.asp
http://www.federalreserve.gov/Pubs/shop/
http://www.thetruthaboutcreditcards.com/get-to-know-your-credit-card-grace-period/

in which consumers can sign exclusive non-linear long-term contracts in period 0, agreeing to a consumption level $c$ as well as a menu of installment plans $(q, r)$ from which self 1 will choose. We define a competitive equilibrium as a set of contracts such that each contract earns zero profits, no contract can generate strictly positive profits, and contracts as well as options in contracts that do not affect expectations or behavior are eliminated. Both as illuminating points of comparison and as possible policy interventions, we also consider markets where contracts are restricted to have a linear structure, with an interest rate determining the cost of shifting more repayment to period 2. In a *long-term restricted* market, firms and consumers can sign exclusive linear contracts in period 0. In a *short-term* market, only one-period contracts are possible. Since this arrangement leaves the period-1 contract choice entirely in self 1's hands, it seems the worst of all.

We begin in Section 4 by assuming that $\beta$ and $\hat{\beta}$ are known to firms, considering first the case of $\hat{\beta} = \beta$, perfect sophistication. Since the borrower correctly predicts her own behavior ex ante, the only contract that survives competitive pressure is the one that maximizes her ex-ante utility: it specifies the optimal consumption level, and commits her to repay in equal installments.

A fully committed repayment schedule, however, is not a competitive equilibrium when consumers are not perfectly sophisticated—when $\hat{\beta} > \beta$. If this was the only contract in the market, a creditor could offer a slightly higher consumption level without increasing either payment, but also include an option for the consumer to delay part of the repayment for a non-trivial fee. Since the borrower is at least slightly more impatient in period 1 than she foresees, the firm can design the penalty to induce unexpected switching, so that the contract is both attractive to the borrower and profitable to the firm. Then, because in equilibrium self 0 expects to choose something else, only self 1 cares about the exact terms of the implemented installment plan, so this option caters *entirely* to self 1's taste for immediate gratification. Even worse, because the consumer believes she will repay quickly, she underestimates the cost of credit, and borrows too much. Hence, any non-sophisticated borrower, *no matter how close to sophisticated*, has discontinuously lower welfare than a sophisticated borrower, and—under a mild assumption on preferences—than she would have even in the short-term market.

Given the above problem, a restricted long-term market often Pareto-dominates the unrestricted

one. By choosing a contract with gross interest rate $1/\beta$, sophisticated borrowers still repay in equal installments and achieve the highest possible long-term utility. More importantly, if a non-sophisticated borrower is not too naive, a linear contract prevents her from seriously mispredicting her future behavior, and hence raises her utility. If many consumers are very naive, a restricted market may have to be combined with other policies to achieve a similar increase in welfare.

To demonstrate the robustness of our main insights, as well as to elaborate on some important issues, in Section 5 we consider equilibria when firms may not know $\beta$ or $\hat{\beta}$. We begin by assuming that $\hat{\beta}$ is observed but $\beta$ is not, and assume that there are two types of borrowers for a given $\hat{\beta}$, one sophisticated ($\beta = \hat{\beta}$) and one non-sophisticated ($\beta < \hat{\beta}$). Then, an equilibrium contract has a low-cost front-loaded repayment schedule ($q > r$) intended for the sophisticated borrower, and a high-cost repayment schedule catered to self 1's taste for immediate gratification intended for the non-sophisticated borrower—features we argue are present in real-life credit-card and mortgage contracts. Confirming the discontinuity in behavior and welfare above, the two options are discretely different even if the non-sophisticated consumer is close to sophisticated.

We also consider how the mixture of sophisticated and non-sophisticated borrowers affects each type's welfare. As in the models of DellaVigna and Malmendier (2004) and Gabaix and Laibson (2006), competitive firms make money on non-sophisticated consumers and lose money on sophisticated consumers. As a result of this cross-subsidy, an increase in the proportion of non-sophisticated consumers always benefits sophisticated consumers, and may benefit non-sophisticated consumers as well. But in our model, sophisticated consumers can also have a positive externality on non-sophisticated ones by "disciplining" firms' exploitative behavior. As the proportion of sophisticated consumers increases, firms shift their strategy from exploiting non-sophisticated consumers toward serving sophisticated consumers, increasing the former's welfare.[4]

Finally, we assume that firms know neither $\beta$ nor $\hat{\beta}$, and impose (roughly) that if a borrower is more optimistic about her future taste (she has a higher $\hat{\beta}$), she tends to be less sophisticated. We

[4] The above cross-subsidy, however, means that restricting the market to be linear has slightly different implications than when $\beta$ is known. Because a restricted long-term market decreases the cross-subsidy, it hurts sophisticated consumers, and hence is not Pareto-improving. But if the non-sophisticated consumer is not too naive, the restricted market still increases population-weighted total welfare because it brings the distorted consumption and repayment terms much closer to optimal.

show that in the unique competitive equilibrium of this market with two-dimensional heterogeneity, firms offer and borrowers accept the same contracts as when $\hat{\beta}$ is observed. Since firms make money on non-sophisticated consumers, they compete more fiercely for consumers with a higher $\hat{\beta}$, offering them better baseline terms. Optimizing given their beliefs, consumers choose the best deal among contracts for which they think they will follow the preferred repayment schedule—which is exactly the contract corresponding to their $\hat{\beta}$. As a result, all non-sophisticated consumers endogenously select a contract under which they do *not* follow the preferred schedule, so that their behavior and welfare is discontinuously different from that of sophisticated consumers.

In Section 6, we consider various extensions and modifications of our framework, including what happens when contracts are not exclusive, when a consumer is overly pessimistic about her future behavior ($\hat{\beta} < \beta$), and when there is heterogeneity in ex-ante preferences. In Section 7, we conclude the paper by emphasizing some shortcomings of our framework, especially the importance of studying two major questions raised by our results: how borrowers' perception of regulation might depend on features of the market, and whether and how consumers might learn about their time inconsistency.

## 2 Related Literature

Our model builds on and is closely related to several recent papers on contracting with time-inconsistent or boundedly rational consumers. While we discuss other differences between these theories and ours below, the most important difference is that we consider a much richer set of welfare implications, and also analyze interventions.

Our paper belongs to the small literature on contracting with time inconsistency, including DellaVigna and Malmendier (2004) on linear contracts and Kőszegi (2005) and Eliaz and Spiegler (2006) on non-linear contracts. Eliaz and Spiegler develop a two-period model in which a monopolist offers contracts in the first period to a population of consumers with homogeneous time-inconsistent preferences about an action to be taken in the second period, but heterogeneous prior probabilities attached to the change in preferences. We modify Eliaz and Spiegler by assuming a different form of naivete about preferences and by focusing on perfect competition, and as a result get

a discontinuity at full sophistication that is not present in their model. We also extend their theory by considering heterogeneity in preferences in addition to beliefs. And we specialize their model to credit markets where time inconsistency derives from a taste for immediate gratification, yielding specific predictions—such as the overborrowing by non-sophisticated consumers and the excessively front-loaded repayment schedules of sophisticated consumers—that would not make immediate sense in their setting.

In a model of contracting over the consumption of harmful and beneficial goods, Kőszegi (2005) asks whether markets are effective at solving the self-control problems arising from time inconsistency (see also Gottlieb 2007 for a related analysis). He shows that there can be a discontinuity in behavior and welfare at full sophistication, but does not ask what happens in credit markets or when a consumer's preferences or beliefs are unobserved, and does not consider interventions similar to ours.

Gabaix and Laibson (2006) model a phenomenon that is both very important for credit markets and generates some results similar to ours. In their model, there is an exogenously given costly add-on (e.g. a printer's cartridge costs or a credit card's fees) that naive consumers might ignore when making purchase decisions, and that sophisticated consumers take costly steps to avoid. Gabaix and Laibson's main finding is that because competitive firms lose money on sophisticated consumers and make money on naive consumers, they may not have an incentive to debias the latter ones. While both forms of naivete are clearly relevant, our focus is on what happens when consumers might misunderstand their *reaction* to a contract rather than the *terms* of the contract. This has the advantage that we can derive borrowers' misprediction of the cost of credit from a general model of consumer preferences and beliefs interacting with profit-maximizing firms—rather than take this as exogenous—and hence to endogenize more features of credit contracts (e.g. an overly front-loaded basic repayment schedule along with a discontinuity afterwards).[5] Because there is (to our knowledge) no theory of what is clear or unclear to consumers, the same would be very difficult for the misunderstanding of contract terms.[6] Finally, there is a major difference between the two

[5] In an extension, Gabaix and Laibson consider a setting where naive consumers underestimate their use of the add-on rather than ignore it completely. Still, the feature of the contract that consumers misunderstand is exogenously given, whereas we derive it endogenously.

[6] Indeed, as Gabaix and Laibson (2006) argue and some studies confirm (Macro International, Inc 2007, for exam-

models in the source of inefficiency: whereas in Gabaix and Laibson's model the welfare loss comes from sophisticated consumers' costly effort to avoid the add-on, in ours it derives largely from the suboptimal contracts naive borrowers receive—an aspect that seems very realistic for credit markets.

Grubb (2007) considers contracting with consumers who overestimate the extent to which they can predict their demand for a product (e.g. their cell-phone usage). To exploit consumers' misprediction, firms convexify the price schedule by selling a number of units at zero marginal price and further units at a positive marginal price. The high marginal price for high amounts of consumption is similar to our basic prediction that moving part of the first installment to later is expensive. Unlike in Grubb (2007), however, in our setting the high price of delaying repayment is imposed as a discontinuous fee, and beyond this fee the marginal price can be low to encourage self 1 to delay more. This feature seems consistent with credit markets; for instance, although a subprime mortgage typically carries a large refinancing penalty, once a borrower pays that penalty there is little extra cost in refinancing more of the mortgage.[7]

# 3 A Model of the Credit Market

## 3.1 Setup

In this section, we introduce our model of a credit market, beginning with borrower behavior. Our formulation of intertemporal preferences is motivated by a time-inconsistent taste for immediate gratification as modeled in Strotz (1956) and Laibson (1997), but is also consistent with a cue-based overweighting of present consumption in the spirit of Bernheim and Rangel (2004b). There are three periods, $t = 0, 1, 2$, with the person setting consumption $c \geq 0$ in period 0 and repaying amounts $q \geq 0$ and $r \geq 0$ in periods 1 and 2, respectively. Self 0's utility is $c - k(q) - k(r)$, and self 1

ple), it is very difficult in practice to define when a contract term is clearly explained, and even to find explanations most consumers will understand. Hence, it is very difficult to eliminate shrouding through regulation. It may be easier to intervene in the ways we consider in this paper, which involve regulating the contract terms themselves.

[7] As we emphasize in Section 6, our model says that the discontinuity in contract terms is closely tied to the time inconsistency of preferences. This might help explain why contracts are discontinuous in credit markets—where time inconsistency is likely to be a big issue—but not for cell phones—where time inconsistency is presumably not such a big issue.

instead maximizes $-k(q) - \beta k(r)$, where $\beta$ satisfying $0 < \beta \leq 1$ parameterizes the time-inconsistent taste for immediate gratification.[8] $k(\cdot)$ is a twice continuously differentiable cost function with $k(0) = 0, 1 > k'(0) \geq 0$, $k''(x) > 0$ for all $x \geq 0$, and $\lim_{x\to\infty} k'(x) = \infty$.

While realistically a taste for immediate gratification applies to all periods, for several reasons we assume that self 0 does not discount the cost of repayment by a factor of $\beta$. Many types of borrowing motivating our analysis—such as first mortgages, home-equity loans for repairs, and credit-card purchases of durable goods—are (largely) for future consumption, and in this case self 0 is likely to weight consumption utility and repayment costs close to equally. Our theory highlights that excessive borrowing and low welfare result even in this case.[9] Of course, if self 0 discounted repayment costs relative to the utility from consumption, the overborrowing would be exacerbated. Even in that case, our specification is useful to isolate how future preferences and beliefs about those preferences affect borrowing and repayment holding current preferences constant.

In line with much of the literature on time inconsistency (DellaVigna and Malmendier 2004, Gruber and Kőszegi 2004, O'Donoghue and Rabin 2006, for example), we equate welfare with long-run, period-0, preferences and put zero weight on self 1's taste for immediate gratification. Although we simplify things by considering a three-period model, in reality time inconsistency seems to be mostly about very immediate gratification that plays out over many short periods. Hence, arguments by O'Donoghue and Rabin (2006) in favor of a long-run perspective apply: in deciding how to weight any particular week of a person's life relative to future weeks, it seems reasonable to snub that week's self—who prefers to greatly downweight the future—in favor of the many earlier selves—who prefer more equal weighting. In addition, the models in Bernheim and Rangel (2004a, 2004b) can be interpreted as saying that a taste for immediate gratification is often a mistake not reflecting true welfare. But while we will throughout this paper discuss welfare from a long-term perspective, other welfare measures may ultimately yield very similar results. For instance, in the basic setting of Section 4, our welfare statements hold in the Pareto sense: due to

[8] None of our results would fundamentally change if the utility from the loan amount $c$ was concave instead of linear. Similarly, since self 1 makes no decision regarding consumption, our analysis would be unaffected if—as would be reasonable with durable goods—the utility from consumption was spread out across periods.

[9] Indeed, although high credit-card borrowing is often invoked as indicating a time-inconsistent taste for immediate consumption, a significant amount of credit-card spending seems to be on durables and other future-oriented goods (Hayhoe, Leach, Turner, Bruin, and Lawrence 2000, Reda 2003).

the overborrowing in period 0, *both* selves 0 and 1 are worse off in the unrestricted market than in the restricted or the short-term one.

The main purpose of this paper is to investigate outcomes as a function of the borrower's sophistication regarding her future taste for immediate gratification. A *sophisticated* self 0 fully realizes that her future preferences will be different from the current ones, so she optimizes taking into account self 1's correctly anticipated behavior. At the other extreme, a *naive* self 0 is completely unaware of her time inconsistency, so she simply takes the period-0 step in maximizing her utility $c - k(q) - k(r)$, incorrectly assuming that self 1 will always take the appropriate next step. To integrate these possibilities in one framework as well as to allow for intermediate levels of naivete, we follow O'Donoghue and Rabin (2001) and suppose that self 0 believes with certainty that self 1 will maximize $-k(q) - \hat{\beta}k(r)$. The parameter $\hat{\beta}$ reflects self 0's beliefs about $\beta$, so that $\hat{\beta} = \beta$ corresponds to perfect sophistication, $\hat{\beta} = 1$ corresponds to complete naivete, and more generally $\hat{\beta}$ is a measure of sophistication.[10] In period 0, the borrower acts to maximize her utility given these beliefs about her future preferences (and hence behavior). In Section 6, we briefly consider how our results would be modified with the other prominent formulation of partial naivete, that of Eliaz and Spiegler (2006) and Asheim (2007).

We think of a group of consumers who are indistinguishable by firms as a separate market, and will define equilibrium for a single separate such market. We assume that the possible $\beta$'s in a market are $\beta_1 < \beta_2 < \cdots < \beta_I$, and $\hat{\beta} \in \{\beta_2, \ldots \beta_I\}$. For any given $\hat{\beta} = \beta_i$, the borrower has $\beta = \beta_i$ with probability $p_i$ and $\beta = \beta_{i-1}$ with probability $1 - p_i$. If firms observe $\hat{\beta}$, then $I = 1$; and if they also observe $\beta$, then in addition $p_i = 0$ or $p_i = 1$. For the case when firms do not observe $\beta$, we will discuss how more general distributions of $\beta$ given $\hat{\beta}$ would affect our results. In addition, in Section 6 we argue that our results do not substantially change if some borrowers are overpessimistic (i.e. have $\hat{\beta} < \beta$) because such borrowers act much like sophisticated ones.

Since the credit market seems relatively competitive—at least at the initial stage of contracting—we assume that the borrowers introduced above interact with competitive, risk-neutral, profit-

[10] While our assumption of degenerate beliefs is analytically very convenient, it is not crucial for the main forces we identify. For instance, our results would not be much affected if a non-sophisticated borrower assigned a small probability to her true $\beta$.

maximizing lenders. Our analysis will make clear that in the first two informational environments we consider—where $\hat{\beta}$ is known—the features and welfare properties of contracts would be the same in a monopoly. The result on the sorting of consumers according to $\hat{\beta}$ in period 0, however, does take advantage of our competitiveness assumption. Firms face an interest rate of zero, and there is no possibility of default.[11] For the bulk of our analysis, we assume that consumers can sign non-linear contracts in period 0 regarding the consumption and repayment schedule, and these contracts are exclusive: once a consumer signs with a firm, she cannot interact with other firms. We discuss the role of full exclusivity in Section 6, arguing that the logic of our results holds under much milder conditions. An *unrestricted credit contract* is defined as an initial consumption $c$ along with a finite menu $C = \{(q_s, r_s)\}_{s \in S}$ of repayment options, and is denoted by $(c, C)$. Note that this specification allows the set of repayment options to be a singleton $\{(q, r)\}$, committing the borrower's future behavior. While in reality such full commitment is of course impossible, we examine this extreme case because it seems to provide the most favorable environment for a market solution to self-control problems to emerge.

As we have mentioned in Section 2, although we believe consumers are in reality often confused to some extent, as a start we assume in this paper that they perfectly understand the terms $(c, C)$ of a credit contract. But they may still be wrong about how they will *respond to* the terms of the contract. The key component of a consumer's evaluation of a contract is an incentive-compatible map:

**Definition 1.** A map $\beta \mapsto (q(\beta), r(\beta)) \in C$ is *incentive compatible* if for each $\beta \in \{\beta_1, \ldots, \beta_I\}$,

$$-k(q(\beta)) - \beta k(r(\beta)) \geq -k(q) - \beta k(r) \text{ for all } (q, r) \in C.$$

A consumer of type $\hat{\beta}, \beta$ believes in period 0 that she will choose $q(\hat{\beta}), r(\hat{\beta})$ from $C$, whereas in reality she chooses $q(\beta), r(\beta)$ if confronted with $C$. This means that when choosing between

[11] For models of market equilibria when consumers can default (but have time-consistent preferences and are rational), see Livshits, MacGee, and Tertilt (2006, 2007). Incorporating plausible reasons for default does not seem to qualitatively affect the logic of our results. If the probability of default is an exogenous and fixed characteristic of the borrower, it presumably does not change discontinuously at full sophistication, so our results would survive essentially unchanged. If the type of switching contract we derive has a role in pushing people into default, it may be less profitable for firms to offer such aggressive contracts. Intuitively, this could weaken the detrimental effect of non-linear contracts on non-sophisticated consumers, the cross-subsidy to sophisticated consumers, and other results, but should not eliminate them.

multiple credit contracts in period 0, the borrower's preferences depend only on $\hat{\beta}$. To enable us to focus on the contracts accepted by consumers, we suppress the strategic interaction between firms and define equilibrium directly in terms of the contracts that survive competitive pressure:[12]

**Definition 2.** A *competitive equilibrium* is a set of contracts $\{(c_i, C_i)\}_{i\in\{2,\ldots,I\}}$ for each $\hat{\beta} \in \{\beta_2, \ldots, \beta_I\}$ and corresponding incentive-compatible maps $q_i(\beta), r_i(\beta)$ with the following properties:

1. [Borrower optimization] For any $\hat{\beta} = \beta_i \in \{\beta_2, \ldots, \beta_I\}$ we have $c_i - q_i(\hat{\beta}) - r_i(\hat{\beta}) \geq c_j - q_j(\hat{\beta}) - r_j(\hat{\beta})$ for all $j \in \{2, \ldots, I\}$.

2. [Competitive market] Each $(c_i, C_i)$ yields zero expected profits.

3. [No profitable deviation] There is no contract $(c', C')$ and incentive-compatible map $q'(\beta), r'(\beta)$ defining perceptions and behavior such that (i) some type with $\hat{\beta} = \beta_i$ strictly prefers $(c', C')$ over $(c_i, C_i)$; and (ii) given all types who strictly prefer $(c', C')$, this contract yields positive expected profits.

4. [Non-redundancy] For each $(c_i, C_i)$ and each installment plan $(q_j, r_j) \in C_i$, there is a type $(\hat{\beta}, \beta)$ with $\hat{\beta} = \beta_i$ such that either $(q_j, r_j) = (q_i(\hat{\beta}), r_i(\hat{\beta}))$ or $(q_j, r_j) = (q_i(\beta), r_i(\beta))$.

Our first requirement for competitive equilibrium is that of borrower optimization: given her predictions about how she would behave with each contract, she chooses her favorite one from the perspective of period 0. Although in principle different borrowers with the same $\hat{\beta}$ may choose different contracts, for simplicity we impose that they do not. Our next two conditions are typical for competitive situations, saying that firms earn zero profits by offering these contracts, and that firms can do no better.[13] The last, non-redundancy condition says that all repayment options in a

[12] This approach is similar in spirit to Rothschild and Stiglitz's (1976) definition of competitive equilibrium with insurance contracts. So long as we impose non-redundancy and that a borrower's perceived behavior with a contract cannot depend on what contracts are available, the same properties emerge in an equilibrium of a simultaneous-move contract-offer game between firms analogous to Bertrand competition in pricing. In addition, although we have not fully worked out whether and how their approach can be translated into our framework, it seems that in all settings in the current paper, a version of Dubey and Geanakoplos's (2002) definition of competitive equilibrium yields the same contracts as our definition. Specifically, think of a repayment contract (in Dubey and Geanakoplos's language, a "pool") as consisting of $C$, with the "price" $c$ of a pool being determined in competitive equilibrium. Consumers can sell contracts and deliver their chosen repayment option, and firms can buy contracts. The key difficulty is in determining the price of an untraded contract. Dubey and Geanakoplos (2002) assume that for any possible contract, there is a vanishingly small share of very profitable consumers buying it.

[13] We could have required a competitive equilibrium to be robust to deviations involving multiple contracts, rather

contract are relevant in that they affect the expectations or behavior of the consumer accepting the contract. Absent this assumption, equilibrium contracts could include unattractive options that no consumer chooses or expects to choose, and which therefore affect neither consumer behavior nor profits. To simplify statements regarding the uniqueness and (relevant) features of the equilibrium contracts, we rule out such meaningless multiplicity by assumption.[14]

For our analysis of the above market with non-linear contracts, we will use two benchmarks in both of which contracts are restricted to have a linear structure. These benchmarks serve both as conceptually useful comparisons for some of our welfare results, and, to the extent that linear contracts generate higher social welfare than more complicated contracts, they can form the basis of a market intervention. In the first benchmark, which we will call the "long-term restricted" market, firms and consumers can still write exclusive contracts in period 0, but the repayment options that are allowed must be linearly related: $q + r/R$ must be a constant for some gross interest rate $R$.[15] This imposes a simple restriction on possible contracts while maintaining the basic timing structure, so it seems like a potentially feasible market intervention. In fact, some states already impose a similar restriction in the payday-loan market, prohibiting charges beyond a single fee capped at a percentage of the loan amount.[16]

In the second benchmark, which we will refer to as a "short-term" market, competitive lenders offer loans in periods 0 and 1 to be repaid in the next period. Clearly, the interest rate in this market will be zero. Because it is both much less feasible and (as will be clear from our analysis)

than the single-contract deviations above. In our specific setting, this makes no difference to the results. This is easiest to see when $\hat{\beta}$ is known: then, offering multiple contracts instead of one cannot help a firm separate different consumers, so it cannot increase profits.

[14] For general distributions of $\beta$ and $\hat{\beta}$, our definition of non-redundancy would have to be a little more inclusive. Specifically, it would have to allow for a repayment schedule $(q_j, r_j) \in C_i$ to be the expected choice from $C_i$ of a consumer type *not* choosing contract $(c_i, C_i)$—because such an option could play a role in *preventing* the consumer from choosing $(c_i, C_i)$. Clearly, this consideration is unimportant if $\hat{\beta}$ is known. Given our assumptions, it is also unimportant when $\hat{\beta}$ is unknown, because the competitive equilibrium in Section 5.3 already fully sorts consumers according to $\hat{\beta}$.

[15] Strictly speaking, we have defined a competitive equilibrium only for the case of unrestricted contracts. When considering the restricted long-term market, one needs to replace the finite set of contracts $\{(c_i, C_i)\}$ with the infinite set of admissible linear contracts.

[16] In Indiana, for example, "no additional amounts may be directly or indirectly charged, contracted for, collected, received, or recovered by the lender" (see http://www.in.gov/dfi/publications/). In particular, beyond the fee imposed on the new loan, lenders cannot charge for renewing a previous payday loan, preventing the kinds of discontinuities in our paper.

worse than the long-term restricted market, we do not think of the short-term market as a policy option. But it is a theoretically interesting extreme case: whereas the long-term restricted market still allows for some commitment through the choice of $R$, a competitive short-term market caters entirely to self 1's preferences in period 1, and hence offers no commitment whatsoever. In this light, our key welfare results are quite surprising: in the unrestricted long-term market, where full commitment to a repayment schedule is feasible, non-sophisticated consumers do worse than in the short-term market, where commitment is impossible.

Of course, there are many possible benchmarks to which we could compare equilibrium outcomes, and, even more importantly, also many plausible market interventions we could contemplate. For instance, because in our model all consumers know their long-term preferences in period 0, requiring them to commit fully to a repayment schedule would maximize long-term utility. This intervention, however, is clearly suboptimal if consumers are subject to ex-post shocks in their financial circumstances. More generally, because our model abstracts from many considerations that would be relevant for policy, doing a full-fledged analysis of optimal policy does not seem very useful. More narrowly, our points below about linear contracts demonstrate merely that eliminating discontinuities in contract terms can be welfare-increasing. This point seems robust to additional features of the credit market missing from our model.

### 3.2 Restating the Problem

As a preliminary step in our analysis, we restate the requirements of a competitive equilibrium (Definition 2) when $\hat{\beta}$ is known in a way that allows us to use contract-theoretic methods throughout the paper. Let $\underline{u}$ be a borrower's perceived utility from the perspective of period 0 if she accepts a purported competitive-equilibrium contract. Given such a market situation, what is the best a firm can do by making its own competing offer? If a borrower accepts our firm's offer, there is a repayment schedule she will expect to choose given her period-0 beliefs $\hat{\beta}$, and there is an option she will actually choose given her true $\beta$. Hence, we can think of the firm as choosing consumption $c$ along with the former "decoy" repayment option and the latter "chosen" repayment options subject to the following constraints. First, for the borrower to be willing to accept the firm's

offer, self 0's utility from the decoy option must be at least $\underline{u}$. This is a version of the standard participation constraint (PC), except that self 0 may make her participation decision based on incorrectly forecasted future behavior. Second, if self 0 is to think that she will choose the decoy option, then given her beliefs $\hat{\beta}$ she must think she will prefer it to the other available options. We call these constraints the perceived-choice constraints (PCC). Third, if a consumer with short-term impatience $\beta$ is to actually choose the repayment schedule intended for her, she has to prefer it to the other repayment options. This is analogous to standard incentive-compatibility constraints (IC) for self 1.

It is clear that a competitive-equilibrium contract must be a solution to the above maximization problem with $\underline{u}$ defined as self 0's perceived utility from accepting the contract: if this was not the case, a firm could solve for the optimal contract and increase $c$ slightly, attracting all consumers and making strictly positive expected profits. In addition, for the solution to the above maximization problem to be a competitive equilibrium, $\underline{u}$ must be such that the highest achievable expected profit is zero. In fact, this is also sufficient:

**Lemma 1.** *Suppose $\hat{\beta}$ is known, and the possible $\beta$'s are $\beta_1, \beta_2 = \hat{\beta}$. The contract with consumption $c$ and repayment options $\{(q(\beta_1), r(\beta_1)), (q(\beta_2), r(\beta_2))\}$ is a competitive equilibrium if and only if there is a $\underline{u}$ such that the contract maximizes expected profits subject to a PC with perceived outside option $\underline{u}$, PCC, and IC, and the maximum profit level is zero.*

Note that this reformulation makes it clear how competition matters when $\hat{\beta}$ is known: through $\underline{u}$. Whether a firm is a monopolist or operates in a competitive market affects its profit-maximization problem only through the consumer's perceived best alternative as summarized by $\underline{u}$. Since for known $\hat{\beta}$ the repayment options of the competitive contract are independent of $\underline{u}$, the degree of competition matters only for determining the consumption level $c$ the consumer is offered.

# 4 Non-Linear Contracting with Known $\beta$ and $\hat{\beta}$

We begin our analysis of non-linear contracting with the case when both $\beta$ and $\hat{\beta}$ are known to firms. We start with the obvious remark that if borrowers are time consistent, the organization of

the credit market does not matter:

**Fact 1.** *If $\beta = \hat{\beta} = 1$, the competitive-equilibrium contract replicates the consumption and repayment outcomes that obtain with the short-term market and the restricted long-term market, and all maximize welfare.*

For the rest of the section, we assume that $\beta < 1$. First, we consider the case of a perfectly sophisticated borrower, for whom $\hat{\beta} = \beta$. Whatever set of repayment options such a borrower is offered, she knows the option she will choose in the end, so any other repayment option is redundant. Applying Lemma 1 and noting that PCC and IC are trivial, a competitive-equilibrium contract solves

$$\begin{aligned} \max_{c,q,r} \quad & q + r - c \\ \text{s.t.} \quad & c - k(q) - k(r) \geq \underline{u}. \end{aligned} \tag{PC}$$

It is clear that PC is satisfied with equality; otherwise, the firm could increase profits by lowering $c$. Plugging PC into the maximand, we can rewrite the firm's problem as

$$\max_{q,r} \quad q + r - k(q) - k(r).$$

Since in a competitive market $c = q + r$, the firm is effectively maximizing self 0's utility. Hence:

**Proposition 1.** *Suppose $\beta$ and $\hat{\beta}$ are known, and $\hat{\beta} = \beta$. Then, the competitive-equilibrium contract has a single repayment option satisfying $k'(q) = k'(r) = 1$, and $c = q + r$. The consumer's utility is strictly higher than in the short-term market.*

To maximize what it can charge a sophisticated consumer, a firm offers self 0—the self who signs the contract—the option that maximizes her utility, including a repayment plan that commits her to repay in two equal installments.

The situation is entirely different for a non-sophisticated borrower, for whom $\hat{\beta} > \beta$. A non-redundant contract consists of a consumption level $c$, a decoy repayment schedule $(\hat{q}, \hat{r})$ self 0 expects to choose, and a repayment schedule $(q, r)$ self 1 actually chooses—with the optimal solution

determining whether $(q, r) = (\hat{q}, \hat{r})$. By Lemma 1, these solve

$$\max_{c,q,r,\hat{q},\hat{r}} \quad q + r - c \tag{1}$$

$$\text{s.t.} \quad c - k(\hat{q}) - k(\hat{r}) \geq \underline{u}, \tag{PC}$$

$$-k(\hat{q}) - \hat{\beta}k(\hat{r}) \geq -k(q) - \hat{\beta}k(r), \tag{PCC}$$

$$-k(q) - \beta k(r) \geq -k(\hat{q}) - \beta k(\hat{r}), \tag{IC}$$

As before, PC binds because otherwise the firm could increase profits by reducing $c$. In addition, IC binds because otherwise the firm could increase profits by increasing $q$. Given that IC binds and $\hat{\beta} > \beta$, PCC is equivalent to $q \leq \hat{q}$: if self 1 is in reality indifferent between two repayment options, then self 0—who overestimates her future patience by at least a little bit—predicts she will prefer the option with more repayment early. Conjecturing that $q \leq \hat{q}$ is optimal even without PCC, we ignore this constraint, and confirm our conjecture in the solution to the relaxed problem below.

The relaxed problem is

$$\max_{c,q,r,\hat{q},\hat{r}} \quad q + r - c$$

$$\text{s.t.} \quad c - k(\hat{q}) - k(\hat{r}) = \underline{u}, \tag{PC}$$

$$-k(q) - \beta k(r) = -k(\hat{q}) - \beta k(\hat{r}). \tag{IC}$$

Notice that in the optimal solution, $\hat{r} = 0$: otherwise, the firm could decrease $k(\hat{r})$ and increase $k(\hat{q})$ by the same amount, leaving PC unaffected and creating slack in IC, allowing it to increase $q$. Using this, we can express $k(q)$ from IC and plug it into PC to get

$$c = k(q) + \beta k(r) + \underline{u}.$$

Plugging $c$ into the firm's maximand leads to Proposition 2 below. Before stating the result, we introduce an assumption we will use to compare welfare in the unrestricted market to that in the short-term market.[17]

[17] Assumption 1 ensures that in the short-term market—which generates a classical consumption-savings problem with hyperbolic discounting analyzed by Laibson (1997)—non-sophisticated individuals borrow more than sophisticated ones, and this and further overborrowing lowers ex-ante utility. Our proofs will make use of this feature, but no other feature of the utility functions in Assumption 1.

**Assumption 1.** One of the following two assumptions holds:

I. [$k$ is a power function.] $k(x) = x^\rho$ with $\rho > 1$.

II. [$k$ is derived from a CRRA utility function with coefficient of relative risk aversion at least 1.] For some $y > 0, \rho > 0$, $k(x) = (y - x)^{-\rho} - y^{-\rho}$.

We are now ready to characterize the competitive-equilibrium contract:

**Proposition 2.** *Suppose $\beta$ and $\hat{\beta} > \beta$ are known. Then, the competitive-equilibrium contract has two repayment options, with the consumer expecting to choose $\hat{q} > 0, \hat{r} = 0$, and actually choosing $q, r$ satisfying $k'(q) = 1, k'(r) = 1/\beta$. Consumption is $c = q + r$, higher than in the short-term market, and higher than that of a sophisticated consumer. The consumer strictly prefers to sign the equilibrium non-linear contract over participating in the short-term market. She has strictly lower welfare than a sophisticated borrower, and if Assumption 1 holds, also than she would have in the short-term market.*

The first important feature of the equilibrium contract is that it is flexible in a way that induces the borrower to unexpectedly change her mind regarding repayment. Intuitively, the flexible contract offers an option for the borrower to repay relatively little, but also introduces an option to delay part of the repayment for a fee. Thinking that she will not use the option, the consumer likes the deal. But since she changes her mind, the firm earns higher profits than with a committed contract.

In addition, since $k'(q) = \beta k'(r)$, self 1's preferences fully determine the allocation of actual repayment across periods 1 and 2, so that the ability to commit perfectly to a repayment schedule does not mitigate the consumer's time inconsistency regarding repayment at all. Intuitively, once the firm designs the contract to induce repayment behavior self 0 does not expect, its goal with the chosen option is to maximize the gains from trade with self 1, so it caters fully to self 1's taste for immediate gratification.

Beyond catering repayment fully to self 1, the equilibrium non-linear contract induces overborrowing in a number of senses. The non-sophisticated consumer borrows more than the sophisticated one; she borrows more than is optimal *given* that repayment is allocated according to self 1's preferences; and she borrows more than she would in the short-term market, and hence (if Assumption

1 holds) has lower welfare than even in that market. To see how the exact level of $c$ is determined, recall that the contract is designed so that self 0 expects to finish her repayment obligations in period 1 ($\hat{r} = 0$). Hence, when deciding whether to participate, self 0 trades off $c$ with $k(\hat{q})$. But from the firm's perspective, $k(\hat{q})$ is just the highest actual total cost of repayment that can be imposed on self 1 so that she is still willing to change the installment plan. This means that the tradeoff determining the profit-maximizing level of borrowing is between $c$ and *self 1's* cost of repayment, which discounts the second installment by $\beta$.

Notice that due to the excessive borrowing in period 0, the non-sophisticated borrower is worse off than the sophisticated one—and worse off than in the short-term market—not only from the perspective of period 0, but also from the perspective of period 1. Indeed, a non-sophisticated borrower repays the same amount in period 1 as a sophisticated borrower, but she repays more in period 2, leading to a lower utility for self 1.[18] Hence, the fact that the borrower is fooled into changing her mind and allocating repayment according to self 1's preferences is ultimately worse for self 1 as well.

It is worth emphasizing that all the above holds for *any* $\hat{\beta} > \beta$—even for an arbitrarily small amount of naivete. Borrowers with $\hat{\beta} \approx \beta$ know that they will be more impatient in the future than they would now like, and in principle have great demand for commitment. Nevertheless, they end up getting none in the market, and borrow too much to boot. There is therefore a discontinuity at full sophistication. In this sense, the equilibrium non-linear contract targets and exaggerates even an arbitrarily small amount of naivete, inducing the consumer to make a non-trivial mistake.[19]

As the above intuition makes clear, non-sophisticated consumers have low welfare in the credit market because firms can offer contracts with discontinuous jumps between what the consumer

[18] The prediction regarding the amount of borrowing contrasts with predictions of hyperbolic discounting in standard consumption-savings problems, such as Laibson (1997). In those problems, as with linear contracts in our setting, whether more naive decisionmakers borrow more depends on the per-period utility function. In our setting, non-sophisticated consumers borrow more than sophisticated ones for any $k(\cdot)$.

[19] While our main interest is in actual behavior, the consumer's anticipated repayment schedule—the decoy option—is also intriguing. The firm attracts borrowers with an inefficiently "virtuous" option: it asks consumers to carry out all repayment in period 1, even though the marginal cost of repaying a little bit in period 2 could be zero. Intuitively, because the decoy repayment terms are never implemented, the firm's goal is not to design them efficiently. Instead, its goal is to attract the consumer while protecting the profits it will later make from her unexpected willingness to abandon the decoy option and delay repayment. Front-loading perceived repayment costs achieves this purpose because self 0 minds early repayment relatively less than does self 1.

expects to choose and what she ends up choosing. We now show that if the consumer is sufficiently sophisticated, a simple intervention mitigates this problem:

**Proposition 3.** *A sophisticated consumer ($\hat{\beta} = \beta$) is equally well off in the restricted long-term market and in the unrestricted long-term market. If a non-sophisticated consumer ($\hat{\beta} > \beta$) is sufficiently sophisticated ($\hat{\beta}$ is sufficiently close to $\beta$), she is strictly better off in the restricted long-term market than in the unrestricted long-term market.*

By counteracting her tendency for immediate gratification as given by $\beta$, a contract with an interest rate of $R = 1/\beta$ aligns self 1's behavior with the borrower's long-run welfare. And since sophisticated borrowers understand their own behavior perfectly, it is profit-maximizing to offer such a contract to them. Hence, for sophisticated consumers the restricted market works just as well as the unrestricted one—they both generate the highest possible level of utility.

More interestingly, restricting contracts to have a linear structure prevents firms from fooling non-sophisticated but not-too-naive borrowers into discretely mispredicting their behavior, and hence raises these consumers' welfare. For any interest rate $R$, a slightly naive borrower mispredicts her future behavior only by a small amount, which leads her to make only a small mistake in how much she borrows. This means that her behavior is very close to a sophisticated borrower's behavior, so that she gets a contract very close to that offered to a sophisticated borrower. As a result, her utility is very close to optimal.

In the case of observable $\beta$ and $\hat{\beta}$, therefore, our intervention satisfies the most stringent criteria of "cautious" or "asymmetric" paternalism (Camerer, Issacharoff, Loewenstein, O'Donoghue, and Rabin 2003): it greatly benefits non-sophisticated borrowers, while it does not hurt sophisticated borrowers. Furthermore, if everyone in the population is rational (sophisticated), the intervention has no effect on outcomes at all.

The linearity of the set of repayment options is not fundamental for the intervention to be welfare-improving. What is important is that the contract should not be able to induce borrowers to make a discrete mistake in predicting their behavior. Any contract in which $r$ is a convex function of $q$ will have this property. For instance, Proposition 3 still holds if we allow contracts with a "baseline" installment plan $\bar{q}, \bar{r}$, a positive interest rate if the borrower repays less than $\bar{q}$ in

period 1, and a zero interest rate if the borrower pays back more than $\bar{q}$ in period 1. Similarly, we could allow linear contracts with bounds on how much can be repaid in period 1.

Proposition 3 is stated for sufficiently sophisticated borrowers because even a linear contract can lead a very naive borrower to severely underpredict how much she will be willing to pay back in period 1, and the firm's attempt to exploit this misprediction can in principle lower welfare below that in the unrestricted market. It is difficult to construct examples where this actually happens; all of our examples use unrealistic utility functions not satisfying Assumption 1. In fact, although we have not found a way to state this formally, it seems that there is a force making it unlikely for linear contracts to hurt even very naive consumers. Namely, the only way to exploit a consumer's underprediction of her future taste for immediate gratification is to set a high interest rate; and a high interest rate in turn induces the consumer to allocate repayment more evenly across periods 1 and 2. This suggests that from a calibrational point of view, the possibility of linear contracts lowering welfare might not be very important.

In addition, even if many consumers are very naive and this leads to linear contracts that yield low welfare, the restricted market can be combined with other regulations to raise welfare above that in the unrestricted market. In particular, a social planner might impose restrictions on how much repayment can be shifted to period 2, or might set an interest-rate cap. Consider the extreme when all consumers are non-sophisticated. Our results above imply that a restricted market with an interest-rate cap of zero already dominates the unrestricted market—simply because it replicates the short-term market. Allowing at least a small positive interest rate is even better, because it induces consumers to repay more of their loan earlier. Of course, if some consumers are sophisticated, an interest-cap can hurt them by preventing them from getting the ex-ante optimal contract. In this case, whether an interest-rate cap is optimal depends on the proportion of sophisticated and non-sophisticated consumers in the population.

## 5 Non-Linear Contracting with Unknown Types

The insights of the previous section may appear to rely on the assumption that firms know $\beta$ and $\hat{\beta}$—and use this information to fool even slightly naive borrowers. This section investigates mar-

ket outcomes when either $\beta$, or both $\beta$ and $\hat{\beta}$, are unobservable to firms. Somewhat surprisingly, our key results on the discontinuity at full sophistication, as well as on welfare-improving interventions, survive. We also show that when different types of borrowers cannot be distinguished, non-sophisticated borrowers cross-subsidize sophisticated ones, and sophisticated borrowers "discipline" firms by preventing very misleading contracts.

Our analysis proceeds from a simple observation: the two-dimensional heterogeneity firms face in our most general setting is unusual because the two dimensions of private information do not affect consumer behavior at the same time. In particular, period-0 behavior is determined solely by $\hat{\beta}$, and (given the available options) period-1 behavior is determined solely by $\beta$. This suggests that consumers may separate according to $\hat{\beta}$ in period 0, and then according to $\beta$ in period 1.[20] Following this reasoning, we begin by investigating outcomes when firms know $\hat{\beta}$ but not $\beta$, and then identify conditions under which the same contracts endogenously separate consumers according to $\hat{\beta}$ in period 0.

## 5.1 Known $\hat{\beta}$, Unknown $\beta$

Suppose that a borrower's $\hat{\beta}$ is known, and she has $\beta_1 < \hat{\beta}$ with probability $p_1$ and $\beta_2 = \hat{\beta}$ with probability $p_2$. For technical convenience, we assume that $k'(0)$ is sufficiently low for first-order conditions throughout the section to describe optimal choices.[21]

Because sophisticated and non-sophisticated borrowers have the same beliefs in period 0, they accept the same contract. Applying Lemma 1, we set up an individual firm's problem as choosing a type-independent consumption $c$ and a menu of type-dependent repayment options $(q_1, r_1), (q_2, r_2)$ subject to participation, incentive, and perceived-choice constraints. As in textbook models of screening (e.g. Bolton and Dewatripont 2005, Chapter 2), we can solve a relaxed problem with only type 1's incentive constraint, and the solution to this problem will also satisfy type 2's incentive

[20] This kind of sorting is reminiscent of sequential screening (Courty and Li 2000, for example), where consumers learn their types in stages, and can be screened sequentially according to what they know at the time. There is also a superficial resemblance in that both sequential screening and our model can generate flexible contracts in equilibrium. But whereas in sequential screening flexibility is a response to consumer types (such as businessmen) with more ex-ante uncertainty regarding their valuation, in our theory it is a way to exploit a borrower's misprediction of her behavior.

[21] For this to be the case for all $p_1 \in (0, 1)$, we must have $k'(0) = 0$. For it to be true for $p_1$ bounded away from 1 ($p_1 \leq a$ for some $a < 1$), $k'(0)$ can be strictly greater than zero.

constraint. Unlike in a standard screening problem, however, both types initially believe that they are type $\beta_2$, so there is only one participation constraint. For the same reason, the perceived-choice constraints are identical to the sophisticated type's incentive constraint, so we do not consider them separately.

Given the above simplifications, the firm's problem is

$$\max_{c,q_1,r_1,q_2,r_2} \quad p_1(q_1+r_1)+p_2(q_2+r_2)-c \tag{2}$$

$$\text{s.t.} \quad c-k(q_2)-k(r_2) \geq \underline{u}, \tag{PC}$$

$$-k(q_1)-\beta_1 k(r_1) \geq -k(q_2)-\beta_1 k(r_2). \tag{IC}$$

In the optimal solution, IC binds; otherwise, the firm could increase $q_1$ without violating IC or PC, increasing profits. In addition, PC binds; otherwise, the firm could increase $q_2$ without violating PC and relaxing IC, again increasing profits. From the binding constraints, we can express $k(q_1)$ and $k(q_2)$:

$$k(q_2) = c-k(r_2)-\underline{u}, \tag{3}$$

$$k(q_1) = k(q_2)+\beta_1(k(r_2)-k(r_1)). \tag{4}$$

Inverting these functions and plugging them into the principal's objective function yields:

**Proposition 4** (Period-1 Screening)**.** *Suppose $\hat{\beta}$ is known, and $\beta$ takes one of two values, $\beta_1 < \hat{\beta}$ or $\beta_2 = \hat{\beta}$, with probabilities $p_1$ and $p_2$, respectively. In the unique competitive equilibrium, the installment plans $(q_1, r_1)$ and $(q_2, r_2)$ satisfy $q_1 < r_1$, $q_2 > r_2$, $q_1 + r_1 > q_2 + r_2$, and*

$$\frac{k'(q_2)}{k'(r_2)}-1 = (1-\beta_1)\cdot\frac{k'(q_2)}{k'(q_1)}\cdot\frac{p_1}{p_2}, \tag{5}$$

$$\frac{k'(q_1)}{k'(r_1)}-\beta_1 = 0. \tag{6}$$

By Equation (5), the sophisticated type's repayment schedule calls for a first installment that is too high even from the long-term perspective of period 0. When considering whether to allocate more of the sophisticated borrower's repayment to period 2, the firm faces a trade-off. On the one

hand, this adjustment increases the sophisticated borrower's period-0 utility, increasing the profits from lending to her. On the other hand, the same adjustment increases the non-sophisticated borrower's period-1 utility and hence lowers her willingness to pay to delay repayment, lowering the profits from lending to her. This tradeoff is similar to that in standard screening problems between increasing efficiency for the less profitable type and decreasing the information rent paid to the more profitable type. In our model, however, the relevant preferences in this tradeoff exist at different times. Since a sophisticated borrower sticks to her ex-ante preferred installment plan, the profit the firm can extract from her depends on period-0 preferences, so this side of the tradeoff takes the period-0 perspective. But since a non-sophisticated borrower abandons her ex-ante preferred installment plan, the profit the firm can extract from her depends on period-1 preferences, so that this side of the tradeoff takes the period-1 perspective. As a result, the sophisticated borrower's schedule is distorted, from the *period-0* perspective, in the opposite direction than the non-sophisticated borrower's *period-1* preferences.

Equation (6) is a version of the "no-distortion-at-the-top" result common to many screening problems. Since type 1 has the highest demand for delaying repayment—so that there is no reason to distort her contract to lower the information rents of more profitable types—she repays in an efficient way from the perspective of period 1.

The difference between the sophisticated and non-sophisticated borrowers' first-order conditions implies a generalization of our insight above that there is a discontinuity in outcomes and welfare at full sophistication, and shows that the discontinuity is now generated by a discontinuity in the terms of a contract both sophisticated and non-sophisticated borrowers sign. In a standard screening model without time inconsistency of preferences, as one type's preference parameter approaches the other type's, the two bundles in the optimal contract converge. As $\beta_1 \nearrow \beta_2$ in our model, however, $q_1$ approaches a number strictly smaller than does $q_2$. In other words, a non-sophisticated borrower, even if she is arbitrarily close to sophisticated, repays in a discontinuously different way from a sophisticated borrower, and is discontinuously worse off as a result.

To confirm the discontinuity in contract terms further, we have solved for properties of the competitive-equilibrium contract when there can be more $\beta$ types given $\hat{\beta}$. Even if the least naive

non-sophisticated borrower is near sophisticated, she often has no possibility to repay in a similar but different way than a sophisticated borrower. And as in Proposition 5 below, it is still true that if non-sophisticated consumers are not too naive, the long-term restricted market yields higher total welfare than the unrestricted market.[22]

The above properties of the competitive-equilibrium contract arguably closely resemble some puzzling features of real-life credit arrangements. Loaded with cash-back bonuses, free rental-car insurance, a grace period, and other perks, the typical credit-card deal is extremely favorable—so long as the consumer does not revolve any debt on her card. If she leaves even $1 unpaid every month, she is charged interest on *all* purchases, and all of a sudden credit-card use becomes quite expensive. Similarly, in-store financing deals often involve no interest for a few months, but if a consumer does not repay fully within the allotted time, she is charged interest on the entire purchase. From a firm's cost perspective, it is unclear why a credit contract should require repayment in full within such a short time horizon, and be so much more expensive otherwise. But a favorable basic deal involving fast repayment, and a discontinuity afterwards, is exactly what our model predicts. Finally, payments in subprime mortgage contracts are often relatively low for a while, but increase drastically thereafter and might conclude with a large "balloon" payment at the end of a relatively short loan period. In combination with hefty prepayment penalties, this structure resembles the key features of contracts in our setting: a fast repayment of the loan to be performed in the future, along with a costly way to switch away from it.[23] As emphasized by Hill and Kozup (2007) and especially Renuart (2004) and as the logic of our model suggests, the high monthly payments or the balloon payment drive borrowers to refinance, and the high prepayment penalty—folded into the principal and financed—serves to make this profitable to the firm. Indeed, in a practice known as "loan flipping," creditors sometimes do this repeatedly (Engel and McCoy 2002).

A weakness of our theory, however, is that it does not convincingly explain why contracts look

[22] The main additional issue relative to our two-type model is that when there are multiple non-sophisticated types, the optimal contract screens between them in period 1. This screening redistributes money from lower-$\beta$ to higher-$\beta$ types, and as a side benefit provides an incentive to repay earlier. Specifically, to lower the information rents paid to the most time-inconsistent borrowers, less time-inconsistent types repay more quickly than is optimal from a period-1 perspective.

[23] Unlike in our model, subprime mortgage contracts do not postulate that refinancing necessarily be carried out by the same lender. As we explain in Section 6, so long as the original firm collects the prepayment penalty, the results are the same.

so different in the prime mortgage market and in the subprime market. Many prime contracts feature very simple installment plans (e.g. the same nominal payment every month for 30 years), and have little or no refinancing charges. To bring our theory in line with these facts, we could assume that borrowers in the prime market are fully sophisticated, but we find this implausible as a primitive assumption. One possibility is that—in contrast to the subprime market—most prime borrowers have a mortgage agent or advisor who helps them procure the loan, and who may help them select like a sophisticated borrower.[24] Of course, our model does not explain why the market is organized this way, and why agents would act in borrowers' long-term interest.[25]

### 5.2 Welfare

In this section, we consider the welfare of borrowers in the above market with $\hat{\beta}$ known and $\beta$ unknown. Our main interest is in comparing welfare to that in the two restricted markets:

**Proposition 5.** *Suppose $\hat{\beta}$ is known, and $\beta$ takes one of two values, $\beta_1 < \hat{\beta}$ or $\beta_2 = \hat{\beta}$. Consumers strictly prefer the contract in the unrestricted market over the contract in the long-term restricted market or participating in the short-term market, and a sophisticated consumer is indeed better off. If Assumption 1 holds, the non-sophisticated consumer has strictly lower welfare than in the short-term market. If the non-sophisticated consumer is sufficiently sophisticated ($\beta_1$ is sufficiently close to $\hat{\beta}$), her welfare, as well as the population-weighted sum of type 1's and type 2's welfare (total welfare), is greater in the long-term restricted market than in the long-term unrestricted market.*

A consumer strictly prefers the long-term unrestricted market over the short-term one, and if she correctly anticipates what she would receive in the restricted market, also over that market. The logic is simple: if this was not the case, a firm could offer a contract replicating the preferred market. Since a sophisticated consumer accurately predicts her utility, she is actually best off in the unrestricted market. In contrast, the discontinuity in welfare between sophisticated and

[24] Many of the same consumers do not get more general financial advise, and so may not act as sophisticated with respect to credit cards.

[25] There are also features completely outside our model that can help explain the difference between the two mortgage markets. For instance, if borrowers in the prime market purchase other financial services from the institutions issuing their mortgage, these institutions may have a stronger reputational incentive not to exploit consumers, as exploited consumers are likely to switch to competitors' products.

non-sophisticated borrowers we have found above is large enough that the latter are worse off than in the short-term market. Hence, even when $\beta$ is unobserved, unrestricted contracts leave non-sophisticated consumers with rather low welfare.

Perhaps most importantly, restricting the long-term market to be linear raises the welfare of non-sophisticated but not-too-naive borrowers. The basic reason is the same as when $\beta$ is known: linear contracts prevent non-sophisticated borrowers from drastically mispredicting their behavior. In the current setting, however, the long-term restricted market makes sophisticated borrowers worse off, so it does not Pareto-dominate the unrestricted one. Intuitively, as was first emphasized by Gabaix and Laibson (2006), because non-sophisticated borrowers are more profitable, in a competitive equilibrium it must be that firms make money on non-sophisticated borrowers and lose money on sophisticated borrowers. Because this cross-subsidy is lower in a restricted market, this intervention lowers the utility of sophisticated borrowers.

When $\beta$ is unknown, therefore, the intervention to restrict contracts to be linear does not satisfy the strict criterion of asymmetric paternalism mentioned above, namely that it should not hurt fully rational consumers in the population. Nevertheless, since it decreases the distortion in the consumption levels and repayment schedules to both types, the restricted market is still socially superior as measured by the population-weighted sum of individuals' welfare. In other words, whatever is the proportion of types in the population, the intervention satisfies a weaker form of asymmetric paternalism: its benefit to non-sophisticated borrowers outweighs its harm to sophisticated borrowers. Furthermore, to the extent that non-sophisticated borrowers are poorer than sophisticated ones, the intervention has redistributive benefits as well in that it decreases the cross-subsidy from less wealthy to wealthy consumers.

In the rest of this section, we analyze how welfare depends on the proportion $p_1$ of non-sophisticated types in the population. The welfare of sophisticated borrowers is strictly increasing in $p_1$:

**Proposition 6** (The Cross-Subsidy Effect)**.** *Suppose $\hat{\beta}$ is known, and $\beta$ takes one of two values, $\beta_1 < \hat{\beta}$ or $\beta_2 = \hat{\beta}$, with probabilities $p_1$ and $p_2$, respectively. The sophisticated type's utility in the competitive equilibrium is strictly increasing in $p_1$.*

The intuition for Proposition 6 is easiest to see based on the cross-subsidies above. Since firms make money on non-sophisticated borrowers, the partial-equilibrium effect of an increase in $p_1$ is an increase in profits. With competition, this extra profit leads firms to compete more fiercely for consumers, forcing them to offer a more attractive-looking option up front. Since sophisticated borrowers correctly anticipate what they will choose, this means that they are better off.

Based on the cross-subsidy effect, it is natural to conjecture that an increase in $p_1$ also benefits non-sophisticated borrowers by lowering the number of borrowers to subsidize and raising the number of borrowers to subsidize them. In our model, however, there is a force acting in the opposite direction. As $p_1$ increases, the mistake non-sophisticated consumers make in estimating their utility increases:

**Proposition 7** (The Discipline Effect)**.** *Suppose $\hat{\beta}$ is known, and $\beta$ takes one of two values, $\beta_1 < \hat{\beta}$ or $\beta_2 = \hat{\beta}$, with probabilities $p_1$ and $p_2$, respectively. Then, $[c - k(q_2) - k(r_2)] - [c - k(q_1) - k(r_1)]$ is increasing in $p_1$.*

Intuitively, a firm can make more money from sophisticated consumers by offering them a more efficient decoy option, but it can make more money from non-sophisticated consumers by increasing the extent to which they misestimate their desire to delay payment and hence their utility. As the number of non-sophisticated consumers increases, the latter consideration becomes more important, hurting non-sophisticated consumers. In this sense, sophisticated consumers provide "discipline" by forcing firms to offer a more reasonable decoy option.[26] In fact, abstracting from the cross-subsidy effect by adjusting $c$ so that firms make zero profits on non-sophisticated borrowers, the welfare of non-sophisticated borrowers is increasing in the proportion of sophisticated borrowers. With the cross-subsidy effect, the impact of a change in $p_1$ on the welfare of non-sophisticated borrowers is ambiguous.

[26] The discipline effect is reminiscent of the positive externality informed consumers exert on uninformed consumers in the pricing models of Salop and Stiglitz (1977) and Wolinsky (1983). Since firms lose informed consumers if they increase premiums, these consumers serve to decrease prices and price dispersion. In contrast to our model, in these theories uninformed consumers have a negative externality on informed consumers.

### 5.3 Unknown $\beta$ and $\hat{\beta}$

We now build on the insights above to identify the competitive equilibrium when $\beta$ and $\hat{\beta}$ are both unobservable to firms. We provide a condition under which consumers self-select according to their perceived preferences $\hat{\beta}$ in period 0, and then according to their true preferences $\beta$ in period 1. This means that even with two-dimensional asymmetric information, all non-sophisticated consumers endogenously select credit contracts in which—to their detriment—they end up changing their mind about repayment.

Let $\underline{u}_i$ be the perceived utility from the competitive-equilibrium contract when $\hat{\beta} = \beta_i$ is observable, with probability $p_i$ the borrower is sophisticated, and with probability $(1 - p_i)$ she is type $\beta_{i-1}$. Our key condition is the following:

**Condition 1.** $\underline{u}_i$ is increasing in $\beta_i$.

Condition 1 states that if $\hat{\beta}$ was observable, the perceived utility from the equilibrium contract would be increasing in $\hat{\beta}$. While this is an endogenous condition, it is intuitively plausible. The condition requires roughly that borrowers who are more optimistic about their future behavior tend to be more naive about it. Since firms compete more fiercely for such profitable consumers, they drive up the perceived attractiveness of the deal. In the current setting with two types of $\beta$ for each $\hat{\beta}$, we require that consumers who believe themselves to be less time-inconsistent are non-sophisticated with sufficiently higher probability.[27]

We argue that under Condition 1, there is a competitive equilibrium in which consumers sign the same contracts as when $\hat{\beta}$ is observed. The crucial part is that from such a set of contracts, consumers self-select according to $\hat{\beta}$ in period 0; then, since there would be no profitable deviation even if firms knew $\hat{\beta}$, there is certainly none in the current setting. There are two parts to our self-selection argument. First, since a borrower of type $\hat{\beta}$ expects to choose the decoy repayment option in a contract intended for any $\hat{\beta}' \leq \hat{\beta}$, among these contracts she prefers the one intended for her simply because (by Condition 1) it gives her the highest perceived period-0 utility. Second,

[27] It is easy to see that Condition 1 is satisfied for some distributions of types. Consider, for instance, a setting with two possible $\hat{\beta}$'s. If the lower $\hat{\beta}$ type is almost certain to be sophisticated while the higher $\hat{\beta}$ type has a non-trivial probability of being non-sophisticated, Condition 1 holds.

while from a period-0 perspective the borrower prefers the decoy option in the contract for $\hat{\beta}' > \hat{\beta}$ to the decoy option in the contract for her own type, she also believes that she will switch away from this option ex post. Once she takes this into account, the period-0 utility from the contract designed for $\hat{\beta}' > \hat{\beta}$ is lower. To see this last point, suppose by contradiction that a type $\hat{\beta}$ preferred to select the contract designed for $\hat{\beta}' > \hat{\beta}$. Then, the contract for $\hat{\beta}$ is suboptimal when $\hat{\beta}$ is known: the contract designed for $\hat{\beta}'$ both attracts $\hat{\beta}$ types and induces all of them to choose the profitable, non-sophisticated repayment option, thereby guaranteeing positive profits when $\hat{\beta}$ is known.

Intuitively, the credit contract intended for a borrower who is more optimistic about her future behavior offers a better deal if the borrower can stick to the most favorable repayment schedule, but requires greater self-control to stick to that schedule. Hence, a consumer takes the most favorable credit contract with which she believes she can still repay according to the ex-ante preferred schedule. This is the contract corresponding exactly to her $\hat{\beta}$.

To illustrate the logic of self-selection through an example, consider a consumer looking to buy a TV on sale financed using store credit that does not accrue interest for six months. The nicer the TV, the sweeter is the deal both because the sale is steeper and because the six-month interest-free period is more valuable. At the same time, it is more difficult to pay back a larger loan in six months. Hence, the consumer chooses the TV which she believes she will just finish paying off in time. But if she is even slightly naive, this TV will be too nice, and she will fail to pay it off.

In fact, the above competitive equilibrium is the unique one:[28]

**Proposition 8** (Period-0 Screening)**.** *Suppose Condition 1 holds. Then, in the unique competitive equilibrium with $\hat{\beta}$ unobserved by firms, each consumer accepts the same contract as when $\hat{\beta}$ is observed by firms.*

Even when firms observe neither the consumer's preferences nor her degree of sophistication, any non-sophisticated consumer—no matter how close she is to sophistication—endogenously selects a contract with which she changes her mind regarding repayment, making her strictly worse off than a sophisticated consumer with the same time-preference parameter $\beta$. Self-selection therefore ensures

[28] The intuition for why the competitive equilibrium must involve full sorting of $\hat{\beta}$'s is the following. If different consumers accepted the same contract in period 0, a competing firm could "steal" and make money on the most valuable type in that pool by offering the optimal screening contract for that type.

that the discontinuity in behavior and welfare we have identified when the firm knows both $\beta$ and $\hat{\beta}$ does not depend on such knowledge.

# 6 Extensions and Modifications

## 6.1 The Role of Exclusivity

Our analysis in this paper relies on the assumption that contracts are exclusive, so consumers cannot borrow from another firm in period 1. We now discuss what might happen if consumers always have access to a competitive credit market.

In the presence of a competitive credit market borrowers have access to in period 1, a sufficient condition for our results is that the original firm can include in the contract a fee—such as the hefty refinancing penalties in predatory mortgages emphasized by Engel and McCoy (2002)—for refinancing with *any* firm in the market, be it the original or the new one. With this fee set to equal the difference between the total payments from the actual and decoy repayment options above, the borrower behaves the same way as with the equilibrium exclusive contract. And since the original firm collects all the profits from the consumer's inclination to change the repayment schedule, our results survive essentially unchanged.

If there is no explicit penalty for contracting with another firm—as is the case with credit cards—the situation is different. Then, in our three-period setting neither committing the consumer nor fooling her into repaying more than expected is possible, because she can always refinance the lowest-cost repayment schedule according to self 1's preferences. But in a more realistic long-horizon setting, borrowers' time inconsistency may prevent them from refinancing. Looking for advantageous refinancing possibilities for existing debt is (for most people) an unpleasant activity, and putting it off by a few days or weeks has a very small cost. As shown by O'Donoghue and Rabin (2001), a sophisticated decisionmaker undertakes such a task relatively soon, but a partially naive time-inconsistent decisionmaker often procrastinates indefinitely, perpetually believing that she will do it at the next opportunity. Consistent with this view, Shui and Ausubel (2004) document that credit-card consumers rarely take advantage of opportunities to lower interest payments on

existing debt. Hence, a lack of exclusivity may hurt sophisticated borrowers without helping non-sophisticated ones.

Finally, even if a borrower refinances a loan, she is likely to do so using a long-term contract of the type we have identified, and if she keeps doing this, she eventually runs out of refinancing options. Indeed, Engel and McCoy (2002) document that predatory mortgages are often refinanced with similarly structured loans, until eventually the borrower cannot obtain any more credit and has to repay according to her existing contract. In the same vein, credit-card balance-transfer deals and teaser rates draw consumers into contracts similar to those they had before, and when the borrower reaches a stage where she can no longer roll over her debt, she has to pay according to the existing contract terms.

### 6.2 Eliaz-Spiegler-Asheim Partial Naivete

To understand the role of the form of partial naivete we have imposed, in this section we briefly consider competitive equilibria with the alternative formulation of Eliaz and Spiegler (2006) and Asheim (2007). Translated into our model, they assume that a consumer's beliefs about her future $\beta$ are binary, assigning probability $p$ to being time-consistent ($\beta = 1$) and probability $1 - p$ to the true $\beta$. In this formulation, it is $p$ that measures the degree of naivete. If $\beta$ and $p$ are observable, a firm's problem can be thought of as choosing $c$, the repayment option $(\hat{q}, \hat{r})$ the borrower thinks she will choose if she is time consistent, and the repayment option $(q, r)$ she correctly thinks she will choose if she has taste for immediate gratification $\beta$. Other than that PC must account for the borrower's probabilistic beliefs and PCC is based on a time-consistent evaluation, the firm's maximization problem is similar to that in (1):

$$\max_{c,q,r,\hat{q},\hat{r}} \quad q + r - c$$

$$c - p\left(k(\hat{q}) + k(\hat{r})\right) - (1-p)\left(k(q) + k(r)\right) \geq \underline{u}, \tag{PC}$$

$$-k(\hat{q}) - k(\hat{r}) \geq -k(q) - k(r) \tag{PCC}$$

$$-k(q) - \beta k(r) \geq -k(\hat{q}) - \beta k(\hat{r}). \tag{IC}$$

Slightly modifying the steps we have used to solve the firm's problem in Section 4 yields that in the optimal contract $k'(q) = 1, k'(r) = 1/(1 - p + p\beta)$, and $\hat{q} > 0, \hat{r} = 0$. In fact, it is easy to show that the equilibrium remains unchanged if the consumer assigns probability $1 - p$ to her true $\beta$ and probability $p$ to any $\hat{\beta} > \beta$ (not necessarily to being time consistent). Furthermore, due to competition the equilibrium contract maximizes the consumer's perceived utility subject to PCC, IC, and the zero-profit constraint $c = q + r$. Since PCC and IC are independent of $p$, this implies that each consumer prefers her equilibrium contract to that of a consumer with a different $p$. Hence, these same contracts sort consumers according to $p$ in period 0, so they constitute a competitive equilibrium even when $p$ is unobserved by firms. This prediction contrasts with the monopoly setting of Eliaz and Spiegler (2006), where the firm chooses not to differentiate slightly naive consumers from sophisticated ones.[29]

Given the above observations, several of our previous insights regarding outcomes and welfare extend to the Eliaz-Spiegler-Asheim formulation of partial naivete. In particular, non-sophisticated consumers are led to delay repayment more often than they expect, and this mistake leads to more borrowing and lower welfare than for sophisticated consumers. In addition, the fact that firms cannot observe consumers' beliefs does not affect the competitive equilibrium at all.

Two of the main points we have made in this paper, however, do not extend to the Eliaz-Spiegler-Asheim model of partial naivete. As is clear from the equilibrium conditions, there is no discontinuity in outcomes and welfare at full sophistication: the borrowed amount is continuously increasing in naivete—with any increase repaid in period 2—and welfare is continuously decreasing in naivete. Relatedly, restricting contracts to be linear does not much affect the welfare of non-sophisticated but not-too-naive consumers. Intuitively, our key welfare predictions rely on the possibility that a non-sophisticated borrower can be confident and wrong in predicting that, *although close*, she will be willing to follow the ex-ante desired repayment schedule. In the Eliaz-Spiegler-Asheim model of partial naivete, a near-sophisticated borrower puts a high probability on what she will actually do, so this kind of situation is impossible.[30]

[29] With this model of partial naivete, we have not found a natural tractable way to introduce heterogeneity in preferences holding beliefs constant.

[30] Using a specification of consumer beliefs as a full distribution over $\beta$ (which subsumes both the O'Donoghue-Rabin and Eliaz-Spiegler-Asheim models as special cases), Heidhues and Kőszegi (2008) show that the probability

Although empirical evidence on the precise form of naivete is lacking, we find the O'Donoghue-Rabin (2001) model of partial naivete sufficiently plausible to focus on its welfare implications in the current paper, especially because the discontinuity prediction it generates is a common feature of credit-card and mortgage contracts. Beyond this point, however, our paper implies that empirical or experimental research examining the *form* of consumers' naivete—not just whether they are naive or even to what degree—would be extremely useful for reaching the most precise welfare and policy conclusions.

### 6.3 The Role of Time Inconsistency

Beyond the form of naivete, a key ingredient in our results is time inconsistency. To see why a misprediction of preferences would not in itself generate discontinuity in behavior and welfare, suppose self 0 discounts period 2 by $\hat{\beta}$, so that she agrees with her perceived future preferences. To modify the firm's problem in (2) to account for this possibility, we only need to change PC to $c - k(\hat{q}) - \hat{\beta}k(\hat{r}) \geq \underline{u}$. It is then easy to establish that to exploit the misprediction of preferences, the optimal contract induces unexpected switching. But if the misprediction is small ($\hat{\beta}$ is close to $\beta$), period-0 and period-1 preferences are close, so the decoy option and the chosen option are also close.

### 6.4 Overly Pessimistic Beliefs

Because it seems most consistent with the evidence on time inconsistency, we have assumed that borrowers tend to be overly optimistic about their future behavior ($\hat{\beta} \geq \beta$). But it is plausible that some borrowers are unrealistically pessimistic about their future preferences, having $\hat{\beta} < \beta$. We briefly discuss what happens in this case under the basic scenario when $\beta$ and $\hat{\beta}$ are known; the logic extends to the other cases as well.

By the same arguments as in Section 4, PC and IC in the firm's maximization problem (1) bind even if $\hat{\beta} < \beta$. If we now ignore PCC, however, we get the same solution as in Section 4,

the consumer assigns to the true $\beta$ or lower is typically crucial in determining her behavior and welfare in a discrete choice problem. If this probability is low, the consumer arranges for too little self-control, and hence later breaks down.

and this solution violates PCC when $\hat{\beta} < \beta$. Hence, PCC must bind. But the only way for both PCC and IC to bind is for the decoy and actual repayment options to be identical—that is, for the contract to be a fully committed contract. And among fully committed contracts, the borrower prefers the one that commits her to repay in equal installments, and that gives her the ex-ante optimal level of consumption. As a result, overly pessimistic borrowers receive the same contract as fully sophisticated borrowers.

There is therefore a fundamental asymmetry between overly optimistic and overly pessimistic beliefs about future preferences: whereas a small amount of overoptimism leads to a discontinuous drop in welfare, a small amount of overpessimism leads to no welfare loss at all. Intuitively, the only way a firm could mislead a pessimistic borrower is by making her think that she will repay less than she actually will. But since the borrower dislikes this possibility, she will be reluctant to sign such a contract. Hence, the best a firm can do is to offer her the optimal committed contract.

Unlike in the case of full sophistication, however, restricting contracts to have a linear structure may affect the welfare of overly pessimistic borrowers. Because these borrowers believe that they need more commitment than they actually do, it is typically not in a firm's interest to offer the interest rate ($R = 1/\beta$) that aligns self 1's behavior with long-run preferences. But by the same argument as in Section 4, for small amounts of pessimism the welfare loss is vanishingly small. Since the welfare gain for overoptimistic borrowers is discrete, if there is even a small fraction of overoptimistic borrowers in the population, linear contracts raise social welfare. For the same reason, our model implies that restricted markets can substantially increase welfare even if borrowers are not only all close to sophisticated, but also on average correct about their future preferences—with some overestimating $\beta$ and some underestimating it.

### 6.5 Heterogeneity in Ex-Ante Preferences

To focus on the role of partially naive time inconsistency in the credit market, we have assumed that borrowers' ex-ante preferences are identical. In reality, consumers of course differ in how much credit they need or want. One way to introduce such heterogeneity is to assume that self 0's utility is $\theta c - k(q) - k(r)$, where $\theta > 0$ is private information. At least in the setting where $\hat{\beta}$ is known and $\theta$

is independent of $\beta$, this modification does not affect our results. Specifically, even in a competitive equilibrium with $\theta$ unobserved by firms, consumers accept the same contracts as when $\theta$ is observed by firms. To see why this is the case, notice that the competitive-equilibrium contract to a type $\theta$ maximizes self 0's perceived utility among contracts that break even in expectation given self 1's actual behavior. Furthermore, since $\theta$ and $\beta$ are independent, the set of these break-even contracts is independent of $\theta$. Hence, a consumer will never prefer the contract intended for a consumer of another $\theta$ type.[31]

# 7 Conclusion

Two major forms of government intervention in economic exchange are competition policy and consumer protection. Whereas competition policy is—at least in principle—heavily informed by theoretical and empirical economic analysis, consumer protection is almost not at all. Quite simply, because economists tend to impose by assumption that most forms of intervention are bad, they have little to contribute to the debate. We do not see why consumer protection could not have rigorous economic foundations just like competition policy. This would help economists inform and improve an already widespread form of intervention in the economy, and—perhaps even more importantly—is likely to make economists more credible when they argue for no intervention.[32] Our paper can be thought of as belonging to the theoretical foundations of such an agenda.

But while it captures some salient features of real-world credit markets and contracts and identifies simple welfare-improving interventions, our setting leaves unanswered a host of further questions about whether and in what way partial naivete justifies intervention. As we have emphasized, the intervention we propose satisfies the criterion of "cautious" or "asymmetric" paternalism (Camerer, Issacharoff, Loewenstein, O'Donoghue, and Rabin 2003) in that it greatly benefits non-sophisticated borrowers and—depending on whether $\beta$ and $\hat{\beta}$ are unobserved—does no harm or

[31] Although we have not considered what modification of Condition 1 we would need, it seems that a version of Proposition 8 also holds in this setting. Even with heterogeneity in $\theta$, consumers will be looking for the best possible deal for which they believe they will repay quickly, so they will tend to sort according to $\hat{\beta}$ in period 0.

[32] In the same spirit, Armstrong (2008) nicely surveys the existing consumer-policy literature from an economic perspective, extends it using economic tools, and identifies the need for further research on the costs and benefits of consumer-protection policies.

less harm to sophisticated borrowers. But in the spirit of "libertarian" paternalism's (Sunstein and Thaler 2003) respect for individual liberty, we can formulate another criterion for interventions: that they should be acceptable to consumers. In our theory, if borrowers anticipated what contracts they would receive in a restricted market, they would be against intervention (see Proposition 5). In some realistic modifications of the model, however, borrowers would prefer—and both sophisticated and non-sophisticated borrowers would strictly benefit from—intervention. In our model, firms redistribute the profits they make on non-sophisticated consumers by making more advantageous offers and hence raising the utility of sophisticated consumers. In a natural variation of our model, firms partly dissipate the high ex-post profits in marketing campaigns aimed at reaching non-sophisticated borrowers, with little or no benefit to sophisticated consumers. Since a restricted market decreases the difference in profits from sophisticated and non-sophisticated borrowers, it decreases or eliminates the incentive for such marketing, and hence generates contracts that borrowers ultimately like better. A little more subtly, if there is no other way to make regulations palatable to everyone, a social planner might take advantage of individuals' lack of sophistication herself—but this time to make a welfare-increasing regulation rather than a welfare-reducing switching contract look good. For example, the social planner might bundle the intervention with a tax subsidy on savings that consumers believe they will take advantage of, but that only sophisticated consumers will actually use. If this subsidy is designed right, the bundled policies can both look attractive to citizens and actually deliver them higher welfare. A more complete analysis of these and related questions is left for future work.[33]

Another important issue we have completely ignored in this paper is the source of consumer beliefs. In particular, consumers may learn about their preferences from their own behavior and that of the firms, and they often seem to have a generic skepticism regarding contract offers even if they do not know how exactly the contract is looking to exploit them.[34] Since our model

[33] A set of questions that requires a much greater departure from our theory than those above is how different proposed policies might be affected by the political process or attempts at enforcement by imperfect institutions. If an in-principle optimal policy will realistically never be adopted, or will be enforced in a way that is harmful for borrowers, it is ultimately a bad policy.

[34] While the issue of learning about time inconsistency has (to our knowledge) not been carefully investigated, evidence by Agarwal, Driscoll, Gaibaix, and Laibson (2007) suggests more broadly that consumers learn from experience to make fewer mistakes in financial decisions, at least until their cognitive abilities start to significantly decline in their 50's.

(like most models of naivete with which we are familiar) starts from exogenously given beliefs, it cannot easily accommodate such learning and meta-sophistication. Nevertheless, our results suggest that becoming more sophisticated can lower welfare, so that learning will not necessarily improve outcomes. So long as a borrower does not become *fully* sophisticated, she switches away from her preferred repayment ex post, so that her increased sophistication does not help her in achieving self-control in repayment. In addition, under Condition 1, her pessimism means that she chooses a worse deal up front, lowering her utility.

## Appendix A: Proofs

**Proof of Lemma 1.** ($\Rightarrow$) Suppose $(c, C)$ satisfies the condition of the lemma. Since only this contract is offered and it satisfies the consumer's participation constraint, it is optimal for her to accept the contract and her choice between contracts is trivial. Thus Condition 1 of Definition 2 is satisfied. Conditions 2 and 4 hold by construction. The key part is to check Condition 3. Consider a contract $(c', C')$ with incentive compatible repayment terms that the consumer strictly prefers. Incentive compatibility guarantees that the contract satisfies IC and PCC, and the fact that the consumer strictly prefers it implies that PC is satisfied when the outside option is $\underline{u}$. Hence, because $(c', C')$ satisfies all constraints that $(c, C)$ does, and $(c, C)$ is optimal given these constraints and yields zero profits, $(c', C')$ cannot yield positive expected profits.

($\Leftarrow$) Since there is only one $\hat{\beta}$ type, there can only be one contract. Let $(c, C)$ be that competitive-equilibrium contract. Condition 4 (non-redundancy) implies that there are only two repayment options in the contract, one for $\beta_1$ and one for $\beta_2$. (These two options are of course identical if there is no $\beta_1$ type.) Incentive compatibility implies that $(c, C)$ satisfies IC and PCC, and it trivially satisfies PC with $\underline{u}$ defined as the perceived utility from $(c, C)$. Now suppose by contradiction that $(c, C)$ does not maximize profits given these constraints. Then, there is a contract $(c', C')$ that satisfies the same constraints and yields strictly positive profits. This means that for a sufficiently small $\epsilon > 0$, $(c' + \epsilon, C')$ attracts consumers and yields strictly positive profits, violating Condition 3 of Definition 2. □

**Proof of Fact 1.** In the short term market the interest rate is zero. Hence a time-consistent agent will maximize her utility by repaying equal amounts in both periods. She thus maximizes $\max_c c - 2k(c/2)$, which implies that $c = 2(k')^{-1}(1)$. It follows from Proposition 1 that she borrows and repays the same amounts in the long-term market, and from the proof of Proposition 3 that she borrows and repays the same amounts in long-term restricted market. □

**Proof of Proposition 1.** In text. □

**Proof of Proposition 2.** We have established in the text that $\hat{q} > 0, \hat{r} = 0, k'(q) = 1, k'(r) = 1/\beta$, and $c = q + r$. Using Proposition 1, the sophisticated and non-sophisticated consumers repay the same amount in period 1, but the non-sophisticated consumer repays more in period 2.

The contract offered to a sophisticated consumer maximizes period-0 welfare among all contracts that break even ($c = q + r$). Since the consumer's contract also breaks even and differs from the

sophisticated one, the consumer is strictly worse off than a sophisticated consumer.

Next, we show that the consumer's consumption is higher than in the short-term market. Recall that the interest rate in a competitive short-term market will be zero. Since self 0 thinks self 1's cost of repayment is $k(q) + \hat{\beta}k(r)$, she believes that for any $c$, self 1 will choose the repayment schedule by minimizing $k(q)+\hat{\beta}k(c-q)$. This minimization problem yields the first-order condition $k'(q) = \hat{\beta}k'(c-q)$. Let $\hat{q}(c)$ denote the unique solution to this first-order condition: this is the amount self 0 thinks self 1 will repay in period 1 if she owes $c$. Note that $\hat{q}(c)$ is a continuously differentiable function of $c$, with a derivative strictly between zero and one.

In choosing how much to borrow in period 0, self 0 solves

$$\max_c \quad c - k(\hat{q}(c)) - k(c - \hat{q}(c)),$$

yielding the first-order condition

$$1 = k'(\hat{q}(c))\hat{q}'(c) + k'(\hat{r}(c))(1 - \hat{q}'(c)).$$

Plugging in $k'(\hat{r}(c)) = k'(\hat{q}(c))/\hat{\beta}$ gives

$$1 = k'(\hat{q}(c))[\hat{q}'(c) + (1 - \hat{q}'(c))/\hat{\beta}].$$

Since the term in square brackets is greater than 1, $k'(\hat{q}(c)) < 1$, which implies that $k'(\hat{r}(c)) < 1/\hat{\beta} < 1/\beta$. These imply that consumption is less than in the unrestricted market.

Now we use the fact that the borrower consumes more than in the short-term market to show that she has lower welfare than in the short-term market. Simple arithmetic yields the following lemma:

**Lemma 2.** *If Assumption 1 holds, in the short-term market $c$ is increasing in $\hat{\beta}$.*

*Proof.* We begin by establishing this for case [I] of Assumption 1. The consumer expects to repay $c$ in a way such that $k'(\hat{q}) = \hat{\beta}k'(c - \hat{q})$, which in case [I] simplifies to

$$\hat{q}(\hat{\beta}, c) = \underbrace{\frac{\hat{\beta}^{\rho-1}}{1 + \hat{\beta}^{\rho-1}}}_{\equiv b(\hat{\beta})} c.$$

Thus, her perceived-period-one utility is $c - (b(\hat{\beta})c)^\rho - ((1 - b(\hat{\beta}))c)^\rho$, which can be rewritten as $c - c^\rho \left[b(\hat{\beta})^\rho + (1 - b(\hat{\beta}))^\rho\right]$. The consumer chooses $c$ to maximize her perceived utility so that $1 = \rho c^{\rho-1} \left[b(\hat{\beta})^\rho + (1 - b(\hat{\beta}))^\rho\right]$. Since $b(\hat{\beta})$ is increasing and less than 1/2, the term in square brackets is decreasing in $\hat{\beta}$, and thus $c$ is increasing in $\hat{\beta}$.

In case [II], let $W \equiv 2y - c$, $s \equiv y - q$, and $t \equiv y - r$. Hence in the short term market $t = W - r$. Rewriting $k'(\hat{q}) = \hat{\beta}k'(c - \hat{q})$, yields

$$s(\hat{\beta}, W) = \underbrace{\frac{\hat{\beta}^{\frac{-1}{1+\rho}}}{1 + \hat{\beta}^{\frac{-1}{1+\rho}}}}_{\equiv b(\hat{\beta})} W.$$

Observe that $b(\hat{\beta})$ is decreasing and greater than 1/2. The consumer's perceived-period-one utility is $c - \Big(b(\hat{\beta})W(c)\Big)^{-\rho} - \Big((1-b(\hat{\beta}))W(c)\Big)^{-\rho}$, which can be rewritten as $c - W(c)^{-\rho}\ [b(\hat{\beta})^{-\rho} + (1 - b(\hat{\beta}))^{-\rho}]$. Since the power function with the exponent $-\rho$ is convex, and $b(\hat{\beta})$ decreasing and greater than 1/2, and increase in $\hat{\beta}$ decreases the term in square brackets. Since at the perceived optimal $c$, $1 = \rho W(c)^{-(\rho+1)}[b(\hat{\beta})^{-\rho} + (1 - b(\hat{\beta}))^{-\rho}]$, and increase in $\hat{\beta}$ must lead to an decrease of $W(c)$ or—in other words—an increase in $c$. □

Consider linear contracts for which repayment is performed according to period-1 preferences. If Assumption 1 holds, repayment amounts in both periods increase linearly in the total amount the consumer needs to repay. If we furthermore restrict attention to contracts in which consumption is equal to total repayments, the consumer's welfare is $c - k(a_1 + bc) - k(a_2 + (1-b)c)$ for some constant $b \in (0, 1)$. Thus, if Assumption 1 holds, among contracts where repayment is performed according to period-1 preferences and consumption is equal to total repayment, the consumer's welfare is single-peaked in consumption. By revealed preference, the maximum occurs at the consumption level that the sophisticated consumer chooses in the short-term market. Lemma 2 implies that a non-sophisticated consumer consumes more in the short-term market than the sophisticated consumer, and she consumes even more than that in the unrestricted market. This implies that she has lower welfare in the unrestricted than in the short-term market.

Finally, we show that the borrower strictly prefers the unrestricted market over the short-term one. To do so, it is sufficient to show that the perceived utility $\underline{u}$ generated by the competitive-equilibrium contract is higher than the consumer's perceived utility in the short-term market. Suppose by contradiction that this is not the case. Then, a single-option contract with the consumption and repayment terms the consumer receives in the short-term market satisfies the conditions of Lemma 1. But this is impossible since the competitive equilibrium we have derived above is unique. □

**Proof of Proposition 3.** Let a linear contract be described by the triplet $(c, R, T)$, where $c$ is consumption, $R$ is the interest rate, and $T$ is the present discounted value of total repayment from the perspective of period 1.

Consider sophisticated borrowers first. Notice that a contract with $R = 1/\beta$ will induce the borrower to repay in equal installments. This means that a contract that combines $R = 1/\beta$ with the ex-ante optimal consumption level $c^*$ and the competitive $T^*$ maximizes the borrower's utility subject to the constraint that consumption is equal to total repayment. Hence, if this contract was not offered but firms made zero profits, for a sufficiently small $\epsilon > 0$ the contract $(c^* - \epsilon, 1/\beta, T^*)$ could be profitably introduced, so that $(c^*, 1/\beta, T^*)$ is the unique competitive-equilibrium contract.

The competitive-equilibrium contract maximizes the consumer's perceived utility among contracts where the firm earns zero profits given the consumer's actual behavior (i.e. $c = q + r$). Consider the set of such contracts; note that it is independent of $\hat{\beta}$. Furthermore, consider the consumer's perceived utility as a function of $(c, R, T)$ over this set. For $\hat{\beta} = \beta$, we have shown above that the function has a unique maximum at $(c^*, 1/\beta, T^*)$. Take any selection of maxima $(c(\hat{\beta}), R(\hat{\beta}), T(\hat{\beta}))$. Notice that given a contract $(c, R, T)$, the borrower's perceived behavior is continuous in $\hat{\beta}$. Hence, as $\hat{\beta} \to \beta$, we must have $(c(\hat{\beta}), R(\hat{\beta}), T(\hat{\beta})) \to (c^*, 1/\beta, T^*)$. This means that in the restricted market the welfare of a non-sophisticated consumer approaches that of a sophis-

ticated consumer as $\hat{\beta} \to \beta$, whereas by Propositions 1 and 2 it does not do so in the unrestricted market. $\square$

**Proof of Proposition 4.** We first establish uniqueness of the competitive equilibrium. By our arguments in the text, the firm's problem reduces to

$$\max_{c,q_1,r_1,q_2,r_2} p_1(q_1+r_1)+p_2(q_2+r_2)-c$$

$$c-k(q_2)-k(r_2)=\underline{u} \tag{PC}$$

$$k(q_2)+\beta_1 k(r_2)=k(q_1)+\beta_1 k(r_1). \tag{IC}$$

Substituting PC into the maximand gives

$$\begin{aligned} \max \quad & p_1(q_1+r_1)+p_2(q_2+r_2)-k(q_2)-k(r_2) \\ k(q_2)+\beta_1 k(r_2) \quad = \quad & k(q_1)+\beta_1 k(r_1) \quad \text{(IC)}. \end{aligned}$$

Let $A=k(q_2), B=k(r_2), D=k(r_1)-k(r_2)$. Then, $k(r_1)=B+D$ and using the IC constraint $k(q_1)=A-\beta_1 D$. Let $f=k^{-1}$. Since $k$ is strictly increasing and strictly convex, $f$ is strictly increasing and strictly concave. Then, the firm's maximization problem can be written as

$$\max_{A,B,D} \; p_1(f(A-\beta_1 D)+f(B+D))+(1-p_1)(f(A)+f(B))-A-B$$

with no constraints. The first-order conditions are:

$$p_1 f'(A-\beta_1 D)+(1-p_1)f'(A)=1 \tag{FOC$_A$}$$

$$p_1 f'(B+D)+(1-p_1)f'(B)=1 \tag{FOC$_B$}$$

$$f'(B+D)-\beta_1 f'(A-\beta_1 D)=0. \tag{FOC$_D$}$$

We show that there is a unique solution to the above system of first-order conditions for $0<p_1<1$. Note that for any $D\geq 0$, there is a unique $A>0$ satisfying (FOC$_A$); call this $\alpha^A(D)$. Since $\alpha^A(D)$ is strictly increasing in $D$, $\alpha^A(D)-\beta_1 D$ must be strictly decreasing in $D$. Also, notice that if $B\geq 0$ is fixed, then for any $D\geq 0$ there is a unique $A$ satisfying (FOC$_D$); call this $\alpha_B^D(D)$. Clearly $\alpha_B^D(D)-\beta_1 D$ is strictly increasing in $D$.

Since $f$ is strictly convex, $f'$ and $f'^{-1}$ are strictly decreasing. Hence if $B\leq f'^{-1}(\beta_1)$, then $f'(B)\geq\beta_1$ and thus $\alpha_B^D(0)=f'^{-1}(f'(B)/\beta_1)\leq f'^{-1}(1)=\alpha^A(0)$. Using the implicit function theorem,

$$\frac{d\alpha_B^D(D)}{dD}=\frac{f''(B+D)+\beta_1^2 f''(\alpha_B^D(D)-\beta_1 D)}{\beta_1 f''(\alpha_B^D(D)-\beta_1 D)}>\beta_1,$$

and

$$\frac{d\alpha^A(D)}{dD}=\beta_1\frac{p_1 f''(\alpha^A(D)-\beta_1 D)}{p_1 f''(\alpha^A(D)-\beta_1 D)+(1-p_1)f''(\alpha^A(D))}<\beta_1.$$

In addition, since $\lim_{y\to\infty} f'(y)=0$, it follows from (FOC$_D$) that as $D\to\infty$, $f'(\alpha_B^D(D)-\beta_1 D)\to 0$ while (FOC$_A$) implies that $f'(\alpha^A(D)-\beta_1 D)>1$. Hence $\alpha_B^D(D)>\alpha^A(D)$ for sufficiently large $D$. Summarizing, since $\alpha_B^D(0)<\alpha^A(0)$, $\alpha_B^D(D)$ is steeper than $\alpha^A(D)$, both curves are continuous, and

for a sufficiently high $D$ we have $\alpha_B^D(D) > \alpha^A(D)$, for this range of $B$ there is a unique $A$ and $D$ satisfying first-order conditions ($\text{FOC}_A$) and ($\text{FOC}_D$). Call these solutions $A^*(B)$ and $D^*(B)$, respectively. If $B > f'^{-1}(\beta_1)$ then $\alpha_B^D(0) > \alpha^A(0)$, and since $\alpha_B^D(D)$ is steeper than $\alpha^A(D)$ no solution to the first-order conditions ($\text{FOC}_A$) and ($\text{FOC}_D$) exists in this range of $B$.

To complete the proof, notice that since $\alpha^A(D)$ is independent of $B$ and $\alpha_D^D(D)$ is increasing in $B$, $A^*(B)$ and $D^*(B)$ are decreasing in $B$; by ($\text{FOC}_A$), this means that $A^*(B) - \beta_1 D^*(B)$ is increasing in $B$, which by ($\text{FOC}_D$) means that $B + D^*(B)$ is increasing in $B$. Hence, the function $p_1 f'(B + D^*(B)) + (1 - p_1) f'(B)$, which is continuous in $B$, is also strictly decreasing in $B$. Furthermore, the function is greater than 1 for $B = 0$, and since for $B = f'^{-1}(\beta_1)$ we have $\beta_1 = f'(B) > f'(B + D^*(B))$, for this value of $B$ the function is less than 1. Hence, there is a unique $B$ for which $B, D^*(B)$ satisfies ($\text{FOC}_B$). Then, $B, A^*(B), D^*(B)$ is the unique solution to the system of first-order conditions. Thus we have shown that the competitive equilibrium is unique.

To characterize the optimal installment plan, we invert equations 3 and 4 and plug them into the principal's objective function, yielding

$$\max_{c,r_1,r_2} \quad p_1 \left[k^{-1}\big(c - k(r_2) - \underline{u} + \beta_1 \left(k(r_2) - k(r_1)\right)\big) + r_1\right] + p_2 \left[k^{-1}\big(c - k(r_2) - \underline{u}\big) + r_2\right] - c. \quad (7)$$

The first-order-conditions with respect to $r_1$ and $r_2$ are:

$$\begin{aligned} p_1 \left[1 - \beta_1 \frac{k'(r_1)}{k'(q_1)}\right] &= 0, \\ p_2 \left[1 - \frac{k'(r_2)}{k'(q_2)}\right] - p_1(1 - \beta_1)\frac{k'(r_2)}{k'(q_1)} &= 0. \end{aligned}$$

Rewriting these first-order conditions gives the equations in the proposition, which in turn imply that $q_1 < r_1$ and $q_2 > r_2$. It remains to establish that $q_1 + r_1 > q_2 + r_2$. Suppose not, then $q_1 + r_1 \leq q_2 + r_2$. If $q_1 + r_1 < q_2 + r_2$, the firm would be better of just offering a single repayment option $(q_2, r_2)$. This option satisfies the PC and, since there is no choice in period 1, it also satisfies PCC and IC, a contradiction. Thus, $q_1 + r_1 = q_2 + r_2$. But in this case the firm would be equally well of when offering a single repayment option $(q_2, r_2)$. This, however, contradicts the fact that in any optimal contract $q_1 < r_1$ and $q_2 > r_2$. □

**Proof of Proposition 5.** First, we show that the borrower strictly prefers the unrestricted market over the short-term (respectively long-term restricted) one. To do so, it is sufficient to show that the perceived utility $\underline{u}$ generated by the competitive-equilibrium contract is higher than the consumer's perceived utility in the short-term (long-term restricted) market. Suppose by contradiction that this is not the case. Then, a contract with the consumption and repayment terms the two types of consumer choose in the short-term (long-term restricted) market satisfies the constraints PC, IC, and PCC in Lemma 1, and breaks even, and is therefore a competitive-equilibrium contract. But this is impossible since a competitive equilibrium identified in Proposition 4 does not replicate outcomes in the short-term (long-term restricted) market: In the short term market, no consumer-type repays more in period 1 than 2. Similarly, for the condition $k'(q_1) = \beta_1 k'(r_1)$ to hold in the long-term restricted market, the firm needs to set $R = 1$. But at this interest rate sophisticated consumers will not repay more in period 1 than 2.

Since sophisticated borrowers understand their behavior, so the fact that their perceived utility is higher than in the short-term or long-term restricted market implies that their actual welfare is also higher.

We now show that for any $0 \leq p_1 < 1$, a non-sophisticated consumer is strictly worse off in the long-term market than a fully naive consumer is in the short-term market. Again, the firm's problem is

$$\begin{aligned} \max \quad & p_1(q_1 + r_1) + p_2(q_2 + r_2) - c \\ c - k(q_2) - k(r_2) &\geq \underline{u} \\ k(q_2) + \beta_1 k(r_2) &\geq k(q_1) + \beta_1 k(r_1). \end{aligned}$$

Recall that for a given $p_1$ the menu of optimal repayment options is independent of $\underline{u}$. Define $\underline{u}(p_1)$ as the level of $\underline{u}$ such that in the solution to the above profit-maximization problem, the firm earns zero profits from type-1 consumers (i.e. $c = q_1 + r_1$). Since $q_1 + r_1 > q_2 + r_2$ by Proposition 4, this would lead firms to lose money on sophisticated consumers. Thus, the competitive-equilibrium contract is the solution to the above with $\underline{u} < \underline{u}(p_1)$. Hence, it is sufficient to prove that in the solution to the above problem with $\underline{u} = \underline{u}(p_1)$, the consumer is strictly worse off than a fully naive consumer in the short-term market.

For $p_1 = 0$, $k'(q_2) = k'(r_2) = 1$, but by (IC) $q_1 + r_1 > q_2 + r_2$. Now because a fully naive consumer thinks she will repay in equal installments, $q_2 + r_2$ is exactly the amount a fully naive consumer borrows in the short-term market. The consumer borrows more here—and when the firm makes zero profits on her she repays exactly what she borrows—so by the same argument as in Proposition 2 she is strictly worse off than the fully naive consumer in the short-term market whenever Assumption 1 holds.

Now notice that $\underline{u}(p_1)$ is decreasing in $p_1$: as $p_1$ increases, in the optimal solution the firm makes more profits on type-1 consumers when holding $\underline{u}$ constant, so to bring those profits back down to zero $\underline{u}(p_1)$ must decrease. By Proposition 7, the difference between the perceived utility $\underline{u}$ and the actual utility of a non-sophisticated consumer is increasing in $p_1$. Combining the above facts and using that optimal repayments for a given $p_1$ are independent of $\underline{u}$, it follows that non-sophisticated consumers' welfare in the contract that solves the above maximization problem for $\underline{u} = \underline{u}(p_1)$ is also decreasing in $p_1$.

To prove that restricted markets increase welfare for sufficiently sophisticated (but still partially naive) consumers, we use a variant of the argument in Proposition 3. As before, the competitive equilibrium contract maximizes the borrowers' perceived utility among contracts that break even. Holding the distribution of consumers fixed, consider the function from $R, T$ to $\underline{u}$ defined in the following way: for any $R, T$, there is a unique consumption level $c$ such that the contract $(c, R, T)$ yields zero profits, which in turn induces a unique perceived utility $\underline{u}$ as a function of $R, T$. For $\beta_1 = \beta_2$, we are back to the case of sophisticated consumers, so that the above function has a unique maximum at $(c^*, 1/\beta_2, T^*)$. Furthermore, since the function is continuous in $\beta_1$, the result follows by the same argument as in Proposition 3.

Next we consider social welfare. As $\beta_1 \to \beta_2$, both types' repayment schedules approach the welfare-optimal one in the long-term restricted market (in which $q_1 = r_1 = q_2 = r_2$). In the long-term unrestricted market $\beta_1 \to \beta_2$, the repayment schedule approaches on in which $k'(q_1) = \beta_2 k'(r_1)$

and is thus inefficient. Thus, for $\beta_1$ sufficiently close to $\beta_2$, social welfare is higher in the long-term unrestricted market in which both types repay optimally. □

**Proof of Proposition 6.** Applying Lemma 1, let $\underline{u}(p_1)$ be the perceived outside option for which the maximum achievable profit level with a proportion $p_1$ of type 1 borrowers is zero; this is also a sophisticated borrowers utility with such a distribution of types. Take any $p_1$ and $p_1' > p_1$. Since the competitive-equilibrium contract with $p_1 < 1$ makes money on non-sophisticated borrowers, if the proportion of $\beta_1$ types is $p_1'$ and the outside option is $\underline{u}(p_1)$, the equilibrium contract with proportion $p_1$ makes positive profits and satisfies the participation and incentive constraints. Hence, we must have $\underline{u}(p_1') > \underline{u}(p_1)$. □

**Proof of Proposition 7.** Notice that $[c - k(q_2) - k(r_2)] - [c - k(q_1) - k(r_1)] = k(q_1) + k(r_1) - [k(q_2) + k(r_2)]$, which using the IC constraint equals $(1 - \beta_1)(k(r_1) - k(r_2)) \geq 0$. Hence, the statement of the proposition is equivalent to $k(r_1) - k(r_2)$ increasing in $p_1$. We prove this statement.

As in proof of Proposition 4, let $A = k(q_2), B = k(r_2), D = k(r_1) - k(r_2)$, which again implies that $k(r_1) = B + D$ and $k(q_1) = A - \beta_1 D$. Defining $f = k^{-1}$, recall that the firm's maximization problem can be written as

$$\max_{A,B,D} \quad p_1(f(A - \beta_1 D) + f(B + D)) + (1 - p_1)(f(A) + f(B)) - A - B$$

with no constraints. The unique solution solves the first-order conditions:

$$p_1 f'(A - \beta_1 D) + (1 - p_1) f'(A) = 1 \tag{FOC$_A$}$$
$$p_1 f'(B + D) + (1 - p_1) f'(B) = 1 \tag{FOC$_B$}$$
$$f'(B + D) - \beta_1 f'(A - \beta_1 D) = 0. \tag{FOC$_D$}$$

Now we show that as $p_1$ increases, in the unique solution $A$ and $D$ increase, and $B$ decreases. Starting from the solution to the above equations for $p_1$, we can define the sequence $A_i, B_i, D_i$ in the following way. First, we choose $A_1$ to satisfy first-order condition (FOC$_A$), then we choose $B_1$ to satisfy (FOC$_B$), and then choose $D_1$ to satisfy (FOC$_D$). We repeat these steps to get $A_2, B_2, D_2$, and so on. It is easy to show that $A_i$ and $D_i$ are increasing sequences, and $B_i$ is decreasing. It is also clear that the sequences $A_i$ and $D_i$ do not diverge: if one of them diverged, (FOC$_A$) implies that the other one would also diverge, which would make it impossible to satisfy (FOC$_D$). Since $B_i$ is bounded from below by zero, all three sequences converge. The limit of these sequences is the solution to the first-order conditions with increased $p_1$. Thus, $D$ is increasing in $p_1$, which establishes the Proposition. □

**Proof of Proposition 8.** We have argued in the text that this is a competitive equilibrium; here, we argue that it is the unique one.

Consider any purported equilibrium in which not all $\hat{\beta}$ types are offered the competitive-equilibrium contract for the case in which $\hat{\beta}$ is known. Let $\underline{u}_i'$ be the perceived utility of $\hat{\beta}_i$ in this situation. First, we show that there is some $i$ such that $\underline{u}_i' < \underline{u}_i$. Suppose by contradiction that $\underline{u}_i' \geq \underline{u}_i$ for all $i$. Then, even if $\hat{\beta}$ was observable, a firm could only break even on each type, and do so only using the competitive-equilibrium contract for each type—a contradiction.

Now consider the highest $i$ such that $\underline{u}'_i < \underline{u}_i$. For a sufficiently small $\epsilon > 0$, a contract that is optimal for type $\hat{\beta}_i$ with the outside option $\underline{u}'_i + \epsilon$ attracts $\hat{\beta}_i$ and makes positive expected profits on this type. Furthermore, since for any $j > i$, $\underline{u}'_i < \underline{u}_i \leq \underline{u}_j \leq \underline{u}'_j$, the contract does not attract $\hat{\beta}_j$. If it attracts $\hat{\beta}_j$ for some $j < i$, it makes strictly positive profits on these consumers, since they all select the non-sophisticated repayment option in the contract. Hence, the contract makes positive expected profits. □

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