Price dispersion and the economics of attention in the information (overload) age

Simon P. Anderson and André de Palma*

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Abstract

We study how the consumer attention span and communication costs affect price competition within and across sectors vying for attention. Equilibrium involves price dispersion within each competing sector, and prices are more dispersed within a sector the more profitable it is and the less attractive are other sectors. A tax on messages can improve welfare.

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1 Introduction

According to a Wiki cite, perhaps the first person to articulate the concept of attention economics was Herbert Simon when he wrote

> ...in an information-rich world, the wealth of information means a dearth of something else: a scarcity of whatever it is that information consumes. What information consumes is rather obvious: it consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it. (Simon 1971, p. 40-41).

*Simon P. Anderson: Department of Economics, University of Virginia, PO Box 400182, Charlottesville VA 22904-4128, USA, sa9w@virginia.edu. André de Palma: Institut Universitaire de France, ThEMA, Université de Cergy-Pontoise, 33 Bd. du Port, 95011, Cergy Cedex, FRANCE. depalma@eco.u-cergy.fr. The first author gratefully acknowledges research funding from the NSF under grant SES-0452864 and from the Bankard Fund at the University of Virginia. We thank conference participants at EARIE 2007 (Valencia) for comments.

Anderson and de Palma (2007) model both the consumer's choice of how much attention to supply and the actions of firms vying for that attention by sending messages advertising their wares. The consumer's attention in this framework is treated as a common property resource insofar as a message sender ignores the effects of its own message on other senders. This means there is a congestion externality, and a tax on messages can improve the allocation of resources.[1] However, one concern with this conclusion was that we assumed in the earlier work that message senders did not compete directly in the marketplace, and only competed for attention. That is, we closed down any direct business-stealing effects in the model. A tax might *a priori* lead to a reduction in price competition by reducing message volume by industry, and the consequences for consumer welfare might be harmful. This is the question we investigate here by specifically modeling competition within each of several industries vying for consumer attention. However, to address the question and focus on the firms, we need to simplify the framework on the consumer side: we now assume the consumer's attention span is exogenously fixed.

The framework we adapt to model the firms actions is adapted from Butters' (1977) seminal work on informative advertising, which is remarkable for delivering a tractable and intuitive description of equilibrium price dispersion. Butters derives a density of advertised prices and sales prices; he proposes a monopolistic competition framework distinct from that of Chamberlin (1933), who is best remembered for his symmetric analysis. In both the Butters and Chamberlinian formulations of monopolistic competition, the competitive part comes from a free-entry zero profit condition that closes the model. In Chamberlin, this means that each producing firm earns zero profits; in Butters there is a zero profit free entry condition for each message sent. The monopolist part of the descriptor in Chamberlin's work comes from heterogeneity of the products sold by firms; in Butters it comes from the market power that firms have due to imperfect information that consumers do not know all firms' prices.

We meld Butters' approach with the advertising clutter approach in van Zandt (2005) and Anderson and de Palma (2007). Reception of messages is passive: the consumer does not search. This corresponds to getting messages from bulk mail, from the television, from billboards, etc. We focus on the interaction of multiple industries competing for individuals' attention. While Butters generates price dispersion because each individual gets only a subset of the price messages, we suppose that the individual screens out some of the messages received. This reflects advertising clutter because an individual is bombarded by too many messages (in "junk" mail, billboards, television, and internet pop-ups) to pay full attention to all of them.

We first characterize an equilibrium model with interaction both within and between industries. Competition within an industry is reflected in the likelihood that a lower price is more likely to be the lowest offer in the set of messages the consumer has screened: nonetheless, high price senders can remain in equilibrium

[1] However, if the consumer's attention is not congested, a tax may worsen the allocation insofar as messages senders do not internalize the consumer surplus from contacting prospective clients.

because of the trade-off between sales probability and mark-up, so all can earn zero profits despite dispersion. Competition between industries arises due to overall competition for a consumer's attention. In this set-up, lower transmission costs in one industry increases messages sent there which congests messages from elsewhere and decreases number of messages sent. This has ambiguous effects on the price dispersion in the other industries. The model also endogenously generates a CES form for the number of messages sent across industries.

The model also generates a different welfare prescription from Butters. While Butters' model has the optimal and equilibrium level of information equal,[2] we find that the market allocation can be improved by taxing messages. This finding reinforces the conclusion of Anderson and de Palma (2007), where the optimal policy in the presence of congestion was a tax on transmission.

2 Message filtering

2.1 Assumptions

There are $\Theta > 1$ active commercial sectors indexed by $\theta = 1, ..., \Theta$.[3] There are M consumers.[4] Firms within each sector produce homogenous goods, and firms transmit an endogenous number of messages n_θ to each consumer for a total number of $N = \sum_{\theta=1}^{\Theta} n_\theta$ messages per consumer. Each message is an advertisement with the price at which the consumer can buy the product from the sending firm. Each consumer examines an exogenous number of messages, ϕ. Individuals cannot tell before examining the messages which came from which sector, so the ϕ messages are drawn at random from the N messages sent. In what follows we will assume for the sake of making some convenient approximations that the total number of messages sent N is large relative to ϕ. This captures advertising clutter / information congestion: each individual examines only a subset of the messages sent.

After a consumer examines the ϕ messages, she chooses the lowest priced one from each sector (we argue below that the probability of ties is zero) and buys q_θ units if that price is no larger than her reservation price for the sector, b_θ.

Each advertiser sends just one message per consumer.[5] The cost per consumer of sending a message from sector θ is γ_θ; this can represent total cost divided by the number of consumers in the case of a billboard message, or the cost of an individual letter for bulk mail, or sending an email offer. The cost

[2] Although see Stegeman (1991) for a demonstration that the market allocation is insufficient when demand has some elasticity. This is because firms then tend to overprice without sufficient regard to the consumer surplus lost.

[3] We later discuss how this number can be endogenously determined.

[4] The number plays no role in the analysis, but will be later useful for extending the model to consumer targeting.

[5] It turns out in the equilibrium below that indeed no sender would want to send a second message: to do so would give a negative profit given the original message just made a zero profit.

of producing the good advertised in the message is c_θ (which is only incurred if the good is bought – think of a pizza for example).[6] The density of messages in a sector at any advertised price is determined by a free-entry condition: we impose a zero-profit condition.

In what follows, we let $\mathbb{P}(p, \theta)$ be the probability of a sale for a message announcing price p in sector θ.

2.2 Solution technique

The probability of the highest-priced sender making a sale to a particular consumer is the probability that the consumer selects its message and no other sender from the same sector is selected. We show below that the equilibrium price dispersion in any sector involves a density of prices with no atoms, and that this density runs up from an endogenous low price to the consumer reservation price for the sector, b_θ.

The solution technique is to equate the probability of making a sale at a particular price as derived from two different angles. The first is determined by the examination technology and the price distribution as the probability that a particular message is the lowest priced one selected from its sector. The second is given by the zero profit condition.

The highest price set by any firm (the consumer reservation price) plays a key role because the only way a sender can make a profit at such a price is if it is the only message drawn from that sector. This ties down the number of messages sent from the sector as a fraction of the total number sent. We then sum over sectors to derive the total number sent, from which we can back out the number in each sector. Armed with that statistic, we can recover the equilibrium price distribution in each sector and its support. This technique will also enable us to determine endogenously the equilibrium number of active sectors.[7]

[6] If the good must be produced beforehand regardless of whether the consumer buys, it suffices to set $c_\theta = 0$ and fold the production cost into the transmission cost, γ_θ.

[7] Butters (1977) supposes M consumers, and a single sending sector. Letters are sent randomly, and each message reaches only a single consumer. Consumers examine all the messages received, and chooses the lowest price received. As with our model, the equilibrium price support has no atoms, no holes, and runs up to b. It starts at $c + \gamma$, because a message at that price is surely read by whoever receives it, and it is a winner (with our model, it must start higher because even the best deal may be unread).

Butters can equate the probability of a sale from two different perspectives, just as we do. The first is the zero profit condition, like ours, $\mathbb{P} = \frac{\gamma}{p-c}$. The second is the finding probability for the price p. For the price b, the likelihood of finding an empty letter box (the only way for the highest price to make a sale) is $\frac{\gamma}{b-c}$. This is thus the fraction of the market unserved, and so is a key statistic in comparing equilibrium to optimum.

The corresponding welfare function is $W = (b-c)MI - \gamma N$ if N messages are sent, where I is the fraction of consumers informed. Hence the optimal number of ads is determined from $(b-c)\, M \frac{dI}{dN} - \gamma$: this equation suggests that an exponential form for the probability of finding an empty letterbox will give equivalence with the equilibrium. This remark underscores the formulation of Butters' letterbox technology. To derive this, note that the probability that at least one of N letters sent reaches a particular one of the M letterboxes is $1 - \left(1 - \frac{1}{M}\right)^N$. When N is large, this is approximately $1 - \exp(-N/M)$ $(= I)$. Hence, $\frac{dI}{dN} = \frac{1}{M} \exp(-N/M) =$

In the sequel we exploit the sequential resolution of the model by first determining (in the next Section) aggregate numbers of messages per sector and total messages, and then describing price dispersion within each sector (in the following Section).

2.3 Message selection probability

We first seek the probability of finding one particular message (from sector θ) and finding no other message from sector θ. The probability that the first message drawn is the one under consideration is $1/N$. The probability that none of the $n_\theta - 1$ others in the sector is chosen on the subsequent $\phi - 1$ draws is $\left(1 - \frac{n_\theta - 1}{N-1}\right)^{\phi-1}$ if these other draws are taken with replacement. Without replacement, and if we assume $\phi << N$, then the probability that the first of the $\phi - 1$ other draws is not from sector θ is $\left(1 - \frac{n_\theta - 1}{N-1}\right) \approx \left(1 - \frac{n_\theta - 1}{N}\right) \approx \left(1 - \frac{n_\theta}{N}\right)$ (for $1 \leq \phi \ll N$). By extension, the probability that *none* of the other $\phi - 1$ messages gets through is:[8]

$$\left(1 - \frac{n_\theta - 1}{N-1}\right)\left(1 - \frac{n_\theta - 1}{N-2}\right) ... \left(1 - \frac{n_\theta - 1}{N - (\phi - 1)}\right) \approx \left(1 - \frac{n_\theta}{N}\right)^{\phi-1}.$$

Therefore, the probability that only the one (of the ϕ) messages is from the sector is:

$$\mathbb{Q}_\theta = \frac{\phi}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1}. \tag{1}$$

This is conveniently rewritten as $\mathbb{Q}_\theta = \frac{\phi}{N - n_\theta}\left(1 - \frac{n_\theta}{N}\right)^{\phi}$, which can also be expressed as[9]

$$\mathbb{Q}_\theta = \frac{\phi}{N - n_\theta}\left(1 - \mathbb{P}_\theta\right),$$

where

$$\mathbb{P}_\theta = 1 - \left(1 - \frac{n_\theta}{N}\right)^{\phi}. \tag{2}$$

$\frac{1}{M}(1 - I)$, from which it follows that the number of uninformed at the optimum is exactly $\frac{M\gamma}{b-c}$, the same as in equilibrium (the business stealing and consumer surplus appropriation externalities net out).

Finally, consider the equilibrium advertised price distribution in the Butters model. By the same logic as for the N letters hitting, if the cumulative NUMBER of letters is $A(p)$ as the probability there is no letter below p, the probability of misses by all lower-priced letters is $\exp\left(-A\left(p\right)/M\right)$ which must equal $\frac{\gamma}{p-c}$ by the zero profit condition. The form of $A\left(p\right)$ and its properties (decreasing, concave) follow directly.

[8] Since there is a large number of messages, drawing one message does not noticably change the residual number of messages in the sector. On the other hand, because ϕ is an integer, one draw does significantly reduce the number of other draws left.

[9] The probability of a single particular message being chosen is here seen as the probability that the sector is not chosen (second term) times the probability that the message is chosen given ϕ draws *outside* the sector (first term). This makes sense once one realizes that the individual message in question, being "small", can just as well be effectively housed initially outside the sector.

Here $\mathbb{P}_\theta$ is readily seen as the probability that there is at least one hit in the sector (θ): the probability that each of the n_θ messages is missed on each of the ϕ draws. The second important link between the two probabilities is that $\mathbb{Q}_\theta = \frac{\partial \mathbb{P}_\theta}{\partial n_\theta}$: only when an extra message in sector θ is the only one received does it raise the chance that the sector is discovered (and therefore the sector probability increases by the probability that a message is the only one from the sector).

3 Advertising levels

3.1 Advertising shares by sector

Consider an advertisement which is sent out for a price equal to the reservation price b_θ. As we argue below, there will be such an ad, and the probability of finding a second ad at the same price is zero. Since $\mathbb{Q}_\theta$ (as given by (1)) is the probability of this ad being the only one found from its sector, the equilibrium zero profit condition means:

$$(b_\theta - c_\theta)\, q_\theta \mathbb{Q}_\theta = \gamma_\theta,$$

where we recall that q_θ is the quantity of good θ demanded (note that demand is rectangular up to reservation price b_θ). Define

$$\pi_\theta = \frac{(b_\theta - c_\theta)\, q_\theta}{\gamma_\theta} > 1,$$

which measures the economic performance of sector θ.[10] Hence the zero profit condition implies that the equilibrium probability the highest-priced sender in the sector θ makes a sale is

$$\mathbb{Q}_\theta = \frac{1}{\pi_\theta}. \tag{3}$$

This probability depends only the intrinsic economic performance index, π_θ, of the sector. Equating this probability, derived from the zero-profit condition, with the probability as derived from the individual's sampling that she gets no other message from the subset in the sector gives the next property.

Lemma 1 *The equilibrium relative sector sizes in message distribution are*

$$\frac{n_\theta}{N} = \left(1 - \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right), \qquad \theta = 1, ..., \Theta. \tag{4}$$

Proof. This follows directly from the equilibrium zero-profit condition $\mathbb{Q}_\theta = \frac{\phi}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1} = \frac{1}{\pi_\theta}$. ■

[10] It is necessary (but not sufficient) that $\pi_\theta = \frac{(b_\theta - c_\theta) q_\theta}{\gamma_\theta}$ exceeds 1 because $(b_\theta - c_\theta)\, q_\theta$ must exceed γ_θ in order for the sender to wish to incur the cost of transmission.

We defer considering the overall comparative static properties of equilibrium because N is still endogenous in the expression above. However, we can use the expression to compare across sectors of different economic characteristics within an equilibrium. Sectors with larger economic weights send more messages because they are more attractive to senders.

Note for future reference that a sector is only viable in equilibrium (from the condition $n_\theta > 0$ in (4)) if $1 > \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)$ or $\pi_\theta > \frac{N}{\phi}$.[11] If this condition does not hold, then even a single message sent from the sector at the highest price would not be expected to cover its costs.

3.2 An IIA property

Sector sizes exhibit a type of "IIA" property (Independence of Irrelevant Alternatives) in the sense that the ratio of market shares of two industries depends only on their profitabilities *for a given N*. However, contrary to the usual IIA property (a characteristic of the multinomial discrete choice model), which stipulates that the ratio of market shares does not change with the number and type of other options, this ratio *does* change here since N changes with the profitability of a third sector. Thus, the standard IIA property does NOT hold for this model.

A related IIA property holds here, with respect to the market shares of competitors ($m_{-i} = \frac{N-n_i}{N}$ where n_i is the number of messages from sector i). From (4), the present IIA property is:[12]

$$\frac{m_{-i}}{m_{-j}} = \left(\frac{\pi_j}{\pi_i}\right)^{\frac{1}{\phi-1}}.$$

This is a property of invariance of the ratio of *all rivals'* advertising levels as the appeal of any rival (outside the pair) changes.

3.3 Aggregate advertising

The next step is to determine the equilibrium market size, N. This comes from summing up the expressions for market shares in (4).

Proposition 2 *The equilibrium total message size is*

$$N = \phi \frac{(\Theta - 1)^{\phi-1}}{\left(\sum_{\theta=1}^{\Theta}\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right)^{\phi-1}}. \tag{5}$$

[11] As we shall see below, this is also the condition for the lowest price in the price support to be below the reservation price. (Since the lowest price is such that the mark-up times the probability of being drawn, which is ϕ/N since a sale is guaranteed for the lowest price in the sector, conditional on being drawn, the condition is, given the low price is critically at b_θ).

[12] It would be interesting to check empirically whether this property is satisfied when the standard IIA property is violated (see Train, 2003, for a discussion of the Hausman test for IIA in the context of discrete choice models).

Proof. Summing up the relations in (4) gives

$$\sum\nolimits_{\theta=1}^{\Theta} n_\theta = N = N\left(\Theta - \sum\nolimits_{\theta=1}^{\Theta}\left(\frac{1}{\phi}\frac{N}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right).$$

Thus $\Theta - 1 = \sum_{\theta=1}^{\Theta}\left(\frac{1}{\phi}\frac{N}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$. Rearranging gives the desired expression. ■

Clearly, from (5), raising the profitability of any sector raises N. So does adding another viable sector.

To see the second point, consider introducing a "barely viable" sector s with $n_s = 0$: by the argument following Lemma 1, the corresponding attractivity of such a new sector s is $\pi_s = N/\phi$. We now verify (as is true by construction of (5)) that introducing this barely viable sector s leaves (5) unchanged. Thence, a viable sector will have a higher $\pi_s > N/\phi$, which will increase N by the first comparative static property noted. To summarize:

$$\begin{aligned}
\frac{N}{\phi} &= \frac{(\Theta-1)^{\phi-1}}{\left(\sum_{\theta=1}^{\Theta}\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right)^{\phi-1}} \\
&= \frac{(\Theta)^{\phi-1}}{\left(\sum_{\theta=1}^{\Theta}\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}} + \left(\frac{\phi}{N}\right)^{\frac{1}{\phi-1}}\right)^{\phi-1}} \\
&< \frac{(\Theta)^{\phi-1}}{\left(\sum_{\theta=1}^{\Theta}\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}} + \left(\frac{1}{\pi_s}\right)^{\frac{1}{\phi-1}}\right)^{\phi-1}}.
\end{aligned}$$

We next consider the symmetric case before going into more detail into the asymmetric one.

3.4 Symmetric case

Under symmetry ($\pi_\theta = \tilde{\pi}$ for all θ), the expression (from (5)) for the total number of messages, N, reduces to

$$N = \phi\left(\frac{\Theta-1}{\Theta}\right)^{\phi-1}\tilde{\pi}. \tag{6}$$

Hence greater attractivity per sector raises linearly the total number of messages sent (and number of messages per sector). As we show below, this is true for the asymmetric case: doubling profitability serves to just double up the number of messages sent at any price. One interpretation of increasing the attractivity per sector is that the communication cost per message falls proportionately. Any such improvement is offset by the rise in messages sent, so all improvements are completely dissipated.

Having more sectors, Θ, raises the total number of messages in a concave fashion, with the number of messages per sector decreasing in the number of sectors if and only if $\Theta < \phi$. Otherwise, with a large number of sectors relative to the number examined, the

Likewise, N increases more slowly than the number of messages examined, ϕ, and consequently $\frac{N}{\phi}$ is falling in ϕ (and clearly the same applies for $n_\theta = \frac{N}{\Theta}$). However, as examples illustrate below, this is not universally true under asymmetry.

3.5 Sector viability

Lemma 3 *Rank sectors by decreasing order of π, and suppose that $\pi_2 > 1$. All sectors with $\pi_\theta \leq 1$ are not viable, and there exists a critical type, $\Theta > 1$ such that all sectors $\theta \leq \Theta$ are active. Equilibrium is unique.*

Proof. From Lemma 1, a sector is active in equilibrium if $1 > \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)$. The current result then follows effectively as a corollary of the argument following Proposition 2. ■

The other important implication of considering viability is that equilibrium congestion effects across sectors may close down a sector. Conversely, a newly entering sector raises the congestion on the others. These we illustrate next.

3.6 Raising a sector's profitability

We noted above that an increase in a sector's profitability will increase the total number of messages sent (see (5)). We now determine what happens to the other sectors. Recall $\frac{n_\theta}{N} = 1 - \left(\frac{1}{\phi}\frac{N}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$ from (4). Hence for an unaffected sector (where π_θ has not changed) it is clear that the sector *share* goes down. However, it is possible the *number* of messages it transmits goes up, as we now show. From (4) we have the derivative

$$\begin{aligned}\frac{dn_\theta}{dN} &= \left(1 - \frac{\phi}{\phi-1}\left(\frac{1}{\phi}\frac{N}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right) \\ &= \left(1 - \frac{\phi}{\phi-1}\frac{(\Theta-1)}{\Theta}\frac{\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}}{\left(\sum_{\theta=1}^{\Theta}\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right)/\Theta}\right)\end{aligned}$$

where we have substituted $\left(\frac{N}{\phi}\right)^{\frac{1}{\phi-1}} = \frac{(\Theta-1)}{\left(\sum_{\theta=1}^{\Theta}\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right)}$ from (5). Define $\chi_\theta = \left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$ and so

$$\frac{dn_\theta}{dN} = \left(1 - \frac{\phi}{\phi-1}\frac{(\Theta-1)}{\Theta}\frac{\chi_\theta}{\bar{\chi}}\right) \tag{7}$$

where $\bar{\chi}$ is the average value of χ_θ.

From a symmetric starting point (where $\chi_\theta = \bar{\chi}$ for all θ), the derivative has the sign of $1 - \frac{\phi}{\phi-1}\frac{(\Theta-1)}{\Theta}$ which is negative if and only if $\phi > \Theta$. If though $\phi < \Theta$, a marginally higher attractivity in one sector causes message numbers to *rise* in all sectors.

In the asymmetric case, (7) indicates that there is a cut-off value of χ_θ for which $\frac{dn_\theta}{dN}$ is negative for higher χ and positive for lower χ. Since π is inversely related to χ, this means that *higher profit sectors are more likely to see an increase* in the number of messages sent.

3.7 Information overflow

The degree of information overflow, N/ϕ, is the extent to which messages are missed. This overflow impacts the degree of competitiveness in the individual markets, and the equilibrium price dispersion per market, as elaborated in the next section. Markets are linked through overall competition for attention. It is more convenient to work with the inverse degree of information overflow, as measured by the ratio: $\xi = \frac{\phi}{N}$. Using expression (5), we get :

$$\xi = \frac{1}{(\Theta-1)^{\phi-1}}\left(\sum\nolimits_{\theta=1}^{\Theta}\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right)^{\phi-1}$$

This is a CES-type function.[13] The function is homogenous of degree -1 in the π_θ: doubling all sector profitabilities will double the total number of messages sent, and so double the overflow rate. Indeed, from (4), the number of messages in each sector doubles. As we shall see below, the message density simply doubles too: the support stays the same. Doubling up (equivalently, halving all communication costs) just dissipates the rents by doubling the number of firms advertising any price.

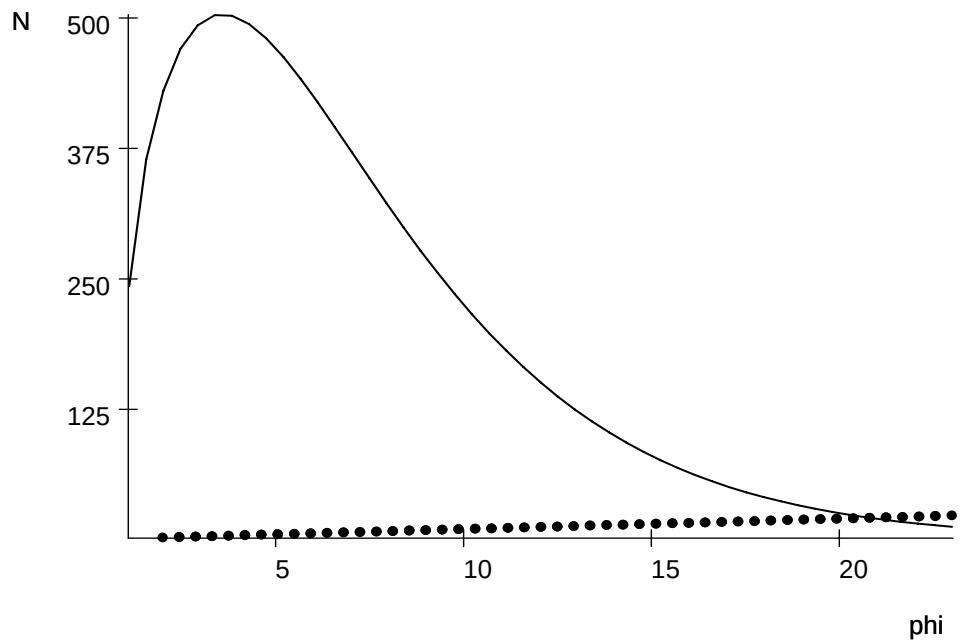
The simulations below show that the number of messages is a quasi-concave function of ϕ, i.e., first increasing, then decreasing with the reception level, ϕ. However, note that it is only increasing for low $\phi < \Theta$ (which is also true under symmetry, as we showed above).[14] More reception has two contradictory consequences. One the one hand, this means that the individual reads more, and more examination in a sector raises the probability of a message from the sector being seen, which raises profitability, and hence the number of messages, ceteris paribus. But it also has the effect of increasing price competition insofar as the price distribution shifts down (as shown below for the symmetric case), as it is more likely a lower price will be found in the sector. This reduces profitability and leads to a smaller number of firms (messages). For low ϕ, the price competition effect is weak in that it is quite unlikely that another message

[13] Therefore, it is maximal at symmetry (under the constraint that the sum of the inverse π_θ's is constant).

[14] The degree of information overflow can also be initially rising with ϕ, but for even smaller values of ϕ.

received will be from the same sector as one already received: extra messages will most likely come from unrepresented sectors. With high reception rates, the price effect dominates.

In the next Figure we plot the equilibrium N as a function of ϕ for the case of 4 sectors, with $\pi_1 = \frac{1}{.002}$, $\pi_2 = \frac{1}{.003}$, $\pi_3 = \frac{1}{.004}$, $\pi_4 = \frac{1}{.005}$. The dotted line is the constraint $N = \phi$.



Total number of messages sent, N, as a function of examination, ϕ.

For low ϕ and given Θ, more examination leads to more messages sent as undiscovered sectors become more likely to be found. For higher ϕ, more examination means more hits in the same sector, which increases price competition and therefore decreases sector attractivity.

4 Equilibrium Price Dispersion

The equilibrium *sales probability* corresponding to a particular price p in sector θ can be determined independently of the other sectors. However, we need to bring in the other sectors to determine which prices are actually used in equilibrium. The equilibrium sales probability is given simply from the zero-profit condition as

$$\mathbb{P}(p,\theta) = \frac{\gamma_\theta}{(p - c_\theta)\, q_\theta} = \frac{\gamma_\theta}{(p - c_\theta)\, q_\theta} \frac{(b_\theta - c_\theta)\, q_\theta}{\gamma_\theta} \frac{1}{\pi_\theta} = \frac{(b_\theta - c_\theta)}{(p - c_\theta)} \frac{1}{\pi_\theta},$$

where $\mathbb{P}(p,\theta) \in (0,1)$ for all p in the support of the equilibrium price distribution.

The equilibrium sales relation above is decreasing and convex in p. We next want to use it to determine the equilibrium *advertised price* distribution. We first argue that the equilibrium advertised price distribution (for any active sector) is a compact interval $\left[\underline{p_\theta}, b_\theta\right]$ with no atoms nor gaps, where the lower bound, $\underline{p_\theta}$, is to be determined below. There are *no atoms* in the price distribution because if there were, any sender choosing the same price as a mass of other senders would raise profits by infinitesimally cutting its price. This would leave its mark-up essentially unchanged but raise sales discretely because it then beats all others at the purported mass point whenever two lowest price messages were the same. The interval has *no gaps* on the support because if there were, the lower price at a gap can be raised leaving the sales probability unchanged but increasing the mark-up. Finally, this same argument implies the support must go up to b_θ: if it stopped short, the highest price firm can raise its price with no penalty on sales probability and increased mark-up.

Lemma 4 *Prices in industry θ are distributed on a compact support $\left[\underline{p_\theta}, b_\theta\right]$ and there are no atoms.*

We now derive the lowest price in the support along with the equilibrium advertised price distribution. Let $F(p,\theta)$ denote the *fraction* of messages in sector θ sent at price p or below. (Then $F\left(\underline{p_\theta},\theta\right) = 0$ and $F(b_\theta,\theta) = 1$). Using the logic above for the sales probability, a message at price p is successful as long as the price is the lowest one received. Using (**??**), we have

$$\mathbb{P}(p,\theta) = \frac{\phi}{N}\left(1 - \frac{n_\theta F(p,\theta)}{N}\right)^{\phi-1},$$

where we simply note that the number of messages sent from the sector with a price no higher than p is $n_\theta F(p,\theta)$.[15] Since $\mathbb{P}(p,\theta) = \frac{(b_\theta - c_\theta)}{(p-c_\theta)}\frac{1}{\pi_\theta}$ by the zero profit condition, we have

$$\frac{(b_\theta - c_\theta)}{(p-c_\theta)}\frac{1}{\pi_\theta} = \frac{\phi}{N}\left(1 - \frac{n_\theta F(p,\theta)}{N}\right)^{\phi-1}, \tag{8}$$

where n_θ/N is given above via (4).

Proposition 5 *The equilibrium advertised price density in sector θ is decreasing and convex on $\left[\underline{p_\theta}, b_\theta\right]$, with distribution given by*

$$F(p,\theta) = \frac{1 - \left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}\left(\frac{(b_\theta - c_\theta)}{(p-c_\theta)}\right)^{\frac{1}{\phi-1}}}{1 - \left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}} \tag{9}$$

[15] Clearly, demamd for the cheapest priced product is ϕ/N for each industry.

and where $\underline{p_\theta}$ is given by

$$\underline{p_\theta} = c_\theta + \left(\frac{N}{\phi}\right)\frac{\gamma_\theta}{q_\theta}, \tag{10}$$

and N is given by (5).

Proof. Since $F\left(\underline{p_\theta}, \theta\right) = 0$ we have $\underline{p_\theta}$ determined by (8) as:

$$\frac{(b_\theta - c_\theta)}{\left(\underline{p_\theta} - c_\theta\right)}\frac{1}{\pi_\theta} = \frac{\phi}{N}.$$

The mark-up of the lowest priced good in sector θ is thence given by (10).

We can now find the equilibrium advertised price distribution from the relation (8) as

$$F(p, \theta) = \frac{N}{n_\theta}\left(1 - \left(\frac{N}{\phi\pi_\theta}\frac{(b_\theta - c_\theta)}{(p - c_\theta)}\right)^{\frac{1}{\phi-1}}\right)$$

Recalling that $\frac{n_\theta}{N} = \left(1 - \left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}\right)$ from (4), we can write (9).

We can readily check that $F(b_\theta, \theta) = 1$ while $F\left(\underline{p_\theta}, \theta\right) = 0$. The corresponding density, $f(p, \theta)$ is strictly positive on $\left[\underline{p_\theta}, b_\theta\right]$, where it is decreasing and convex (as shown by differentiation of (9)). ■

As in Butters (1977), lower prices are advertised more heavily. In the Butters model, the corresponding lowest price, $\underline{p}$, would be simply $c_\theta + \frac{\gamma_\theta}{q_\theta}$ (allowing for the quantity effect, which Butters does not have) because such a price just covers the cost of production plus sending the message. In contrast to the Butters model, here the lowest price in any sector does not garner a sale with probability 1. In the Butters version, the lowest price must always get a sale because there is no information congestion, and no possibility that the message remains unread. Here, information overflow pushes up the lowest price in the support: this is needed to compensate for the likelihood that the message may not be received.

The intuition for the expression for the lowest price in the support is straightforward. A message sent at this lowest price always beats all the other messages from the sector. Hence the sales probability is just the probability that it is read at all, which is simply $\frac{\phi}{N}$ since it has ϕ shots and is in a pool of N messages. Equating this probability times the mark-up to the cost of sending the message gives the resulting expression.

Changes within the sector affect the support as well as the aggregate message volume N. A sector can become inviable if it faces tough competition from other sectors and/or it is quite unattractive itself. Viability can be expressed as the condition that the price support not collapse. That is $\underline{p_\theta} < b_\theta$. Writing out the condition, it means that $\frac{N}{\phi} < \pi_\theta$; this is the same condition from (4) for $n_\theta > 0$ in equilibrium.

In the symmetric case, N is given by (6) as $N = \phi\left(\frac{\Theta-1}{\Theta}\right)^{\phi-1}\tilde{\pi}$, and so (9) becomes

$$F(p,\theta) = \Theta\left(1 - \frac{(\Theta-1)}{\Theta}\left(\frac{(b-c)}{(p-c)}\right)^{\frac{1}{\phi-1}}\right), \quad \text{for } p \in \left[\underline{p}, b\right]$$

where $\underline{p} = c + \left(\frac{\Theta-1}{\Theta}\right)^{\phi-1}(b-c)$ (by (10)).[16] Hence, as ϕ rises, the lower bound $\underline{p}$ falls, and so intra-sector competition rises in this respect. Indeed, $F(p,\theta,\phi_1) > F(p,\theta,\phi_2)$ as $\left(\frac{(b-c)}{(p-c)}\right)^{\frac{1}{\phi_1-1}} < \left(\frac{(b-c)}{(p-c)}\right)^{\frac{1}{\phi_2-1}}$ or $\phi_2 < \phi_1$. Thus a *higher examination rate leads to more price competition* in the sense of First Order Stochastic Dominance. We consider further FOSD relations in the next section.

4.1 Dispersion

The simplest measure of price dispersion is the breadth of the support of the equilibrium prices. This is $b_\theta - \underline{p_\theta}$, where $\underline{p_\theta} = c_\theta + \left(\frac{N}{\phi}\right)\frac{\gamma_\theta}{q_\theta}$, from above. Ceteris paribus, dispersion is smaller the greater is $\left(\frac{N}{\phi}\right)\frac{\gamma_\theta}{q_\theta}$.

Greater sector profitability impacts the affected sector by increasing price competition, but reduces price competition in other sectors because their messages are crowded out. Nonetheless, the number of messages sent in other sectors can actually rise (see the earlier analysis of n_θ) because the reduced price competition can raise profits per firm (which then must be reduced by further entry).

Proposition 6 *An increase in the attractivity of one sector decreases prices in that sector and increases prices in the other sectors, in the sense of first-order stochastic dominance. A proportional increase in the attractivity of all sectors leaves the price distribution unchanged.*

Proof. Recall $F(p,\theta) = \frac{1-\left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}\left(\frac{(b_\theta-c_\theta)}{(p-c_\theta)}\right)^{\frac{1}{\phi-1}}}{1-\left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}}$ by (9). Hence $\frac{dF}{d\pi_{\theta'}}$ (for $\theta' \neq \theta$) has the opposite sign from $\frac{dN}{d\pi_{\theta'}}$, which is positive, as already established. Hence $F(p,\theta)$ decreases in $\pi_{\theta'}$. However, $\frac{dF}{d\pi_\theta}$ has the opposite sign, since $\frac{N}{\pi_\theta}$ is decreasing in π_θ (from (5)). Hence, $F(p,\theta)$ increases in π_θ.

If all sectors increase proportionately in attractivity, $\frac{N}{\pi_\theta}$ is unchanged (by the homogeneity property noted earlier) and so $F(p,\theta)$ is unchanged. ■

This means that advertised prices (and price dispersion) can be negatively correlated across sectors. If one sector becomes more desirable, prices fall as

[16] As $\left(\frac{\Theta-1}{\Theta}\right)^{\phi-1}\tilde{\pi} \to 1$, then $N \to \phi$, and, (by (10)), $\underline{p}$ goes to $c + \frac{\gamma}{q}$: this is like the Butters price.

competition intensifies. But the more intensive message volume crowds out messages from other sectors, and so relaxes competition in those other sectors. On the other hand, across-the-board changes affecting all sectors can leave prices the same. The last property underlies the result in the next Section that benefits from proportionately lower message transmission costs are dissipated fully: equivalently, a (proportional) tax might be raised without deadweight loss.

4.2 Sales Price Distribution

The *sales price distribution* differs from the advertised price distribution because lower prices are more likely to get sales, and also because even the lowest advertised price does not always make a connection. The advertised price distribution is $F(p,\theta)$ as given in Proposition 5; denote the corresponding density by $f(p,\theta)$. Then the sales price density at p is the advertised price density, $f(p,\theta)$, times the probability, $\mathbb{P}(p,\theta)$, that the advertised price makes a sale (up to a multiplicative constant, k_1). Since the latter is $\frac{\gamma_\theta}{p-c}$, this means that the sales price density is:

$$g(p,\theta) = k_1 f(p,\theta)\,\mathbb{P}(p,\theta) = k_2\left(\frac{1}{p-c_\theta}\right)^{\frac{1}{\phi-1}+2}$$

where k_2 is a constant to be determined. The corresponding cumulative distribution is

$$G(p,\theta) = k_3\left[\left(\frac{1}{\underline{p}-c_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{1}{p-c_\theta}\right)^{\frac{1}{\phi-1}+1}\right].$$

The constant k_3 can be determined from the relation $G(b_\theta,\theta)=1$. Differentiating gives the following properties.

Proposition 7 *The equilibrium sales price distribution in sector θ is decreasing and convex on $\left[\underline{p_\theta}, b_\theta\right]$, with distribution given by*

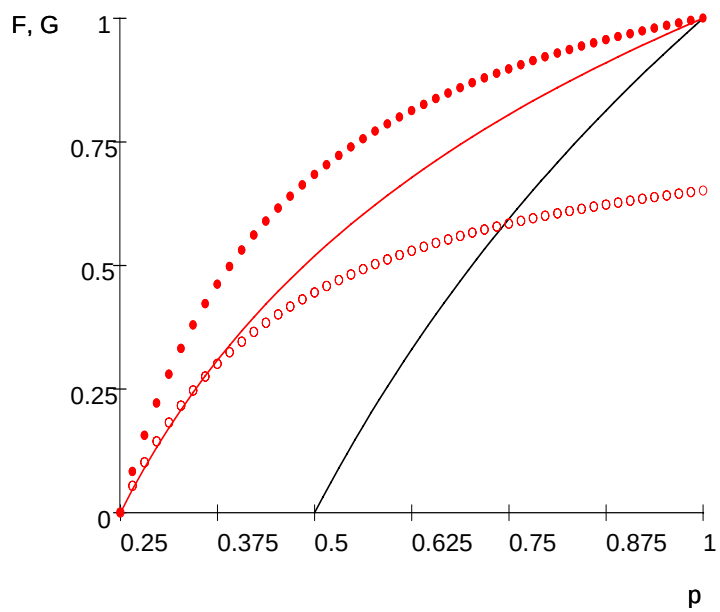
$$G(p,\theta) = \frac{\left(\frac{1}{\underline{p_\theta}-c_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{1}{p-c_\theta}\right)^{\frac{1}{\phi-1}+1}}{\left(\frac{1}{\underline{p_\theta}-c_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{1}{b_\theta-c_\theta}\right)^{\frac{1}{\phi-1}+1}}. \tag{11}$$

where $\underline{p_\theta}$ is given by (10).

Note that this is the distribution of the actual transaction prices for the sector, i.e., conditional on a sale being made. Since the probability of a sale being made in the sector is $\mathbb{P}_\theta = 1-\left(1-\frac{n_\theta}{N}\right)^\phi$, then $\mathbb{P}_\theta G(p,\theta)$ represents the probability of a sale being made in sector θ at a price below p. This statistic allows us to calculate the expected consumer surplus from the sector.

The next Figure shows the difference: the advertised price distribution is given as the red solid line. (The parameter values used are: $b=1$, $c=0$,

$N/\phi = 10$, $\phi = 10$ and $\gamma = 0.025$.)[17] The distribution of actual transactions conditional on a sale being made, $G(p, \theta)$, is the dotted red line. The red circles represent the cumulative distribution of actual sales. This sales price distribution lies above the advertised price one for low p. This means simply that more sales are made at low prices (as high priced ads are beaten). This must happen throughout the range in Butters' model because each ad is received by someone. However, in our context, there is a probability that ads are not received at all. This feature is reflected in the fact that $G(b) < 1$ in our model (see Figure): the consumer may get no ads from the sector and no sale is made at all. Finally, the Figure also shows (in black) the advertised price distribution for $\gamma = 0.05$.



Advertised price functions (solid) and sales price distributions.

We can use $G(p, \theta)$ to calculate the *size distribution of firms* within sector θ. In particular, we can simply replace $p = c_\theta + \frac{\gamma_\theta}{Q}$ (from the zero profit condition) where $Q = \mathbb{P}q_\theta$ is the size of the firm, in terms of units sold. Substituting in (11) gives:

$$H(Q, \theta) = 1 - G\left(c_\theta + \frac{\gamma_\theta}{Q}, \theta\right) = \left[\frac{\left(\frac{Q}{\gamma_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{Q_L}{\gamma_\theta}\right)^{\frac{1}{\phi-1}+1}}{\left(\frac{Q_H}{\gamma_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{Q_L}{\gamma_\theta}\right)^{\frac{1}{\phi-1}+1}}\right],$$

[17] The value $N/\phi = 10$ can be consistent with a specific number of other sectors.

where Q_H $(= \frac{\gamma_\theta}{\underline{p_\theta} - c_\theta})$ is the highest output (corresponding to the lowest price, $\underline{p_\theta}$) and Q_L $(= \frac{\gamma_\theta}{b_\theta - c_\theta})$ is the lowest output (corresponding to the highest price, b_θ). The corresponding density, $h(Q, \theta)$, is proportional to $Q^{\frac{1}{\phi - 1}}$ and so is increasing, and concave for $\phi > 2$. Note that aggregate congestion interactions DO enter here (and are simply described through the N effect), since $\underline{p_\theta} = c_\theta + \left(\frac{N}{\phi}\right) \frac{\gamma_\theta}{q_\theta}$ enters into Q_H $(= \frac{\gamma_\theta}{\underline{p_\theta} - c_\theta})$.

5 Normative properties

The welfare function is given by summing over sectors the total sector surplus times the probability a sale is made in the sector, and then subtracting the message costs. Recall that $\mathbb{P}_\theta$ is the probability of at least one hit in sector θ, and write this as $\mathbb{P}_\theta(n_\theta, N) = 1 - \left(1 - \frac{n_\theta}{N}\right)^\phi$, which depends only on own message number and the aggregate. Let n denote the vector $n_1, ..., n_\Theta$; then we can write the form of the welfare function as

$$W(n, N) = \sum\nolimits_{\theta=1}^{\Theta} \left[(b_\theta - c_\theta) q_\theta \mathbb{P}_\theta(n_\theta, N) - \gamma_\theta n_\theta\right]. \tag{12}$$

This form is convenient for the demonstration that follows.

Proposition 8 *The allocation of messages across sectors is optimal given the number of messages transmitted at the equilibrium. However, the total number of messages transmitted is excessive.*

Proof. Let N be given at the equilibrium level stipulated by (5), $N = N^e$, and we wish to show that the division of these messages effectuated in equilibrium is optimal. Optimal choice of the message levels by sector under the constraint that they must sum to the equilibrium level, N^e, implies that the welfare derivatives with respect to the individual n_θ's must be equalized. Since we have $\frac{dW}{dn_\theta} = \frac{\partial W}{\partial n_\theta} + \frac{\partial W}{\partial N}$, it follows that the partial derivatives, $\frac{\partial W}{\partial n_\theta}$, are to be equalized, and the corresponding n_θ's must sum to N^e.

Now, from (12), we have that

$$\begin{aligned} \frac{\partial W}{\partial n_\theta} &= (b_\theta - c_\theta) q_\theta \frac{\partial \mathbb{P}_\theta(n_\theta, N)}{\partial n_\theta} - \gamma_\theta \\ &= (b_\theta - c_\theta) q_\theta \frac{\phi}{N} \left(1 - \frac{n_\theta}{N}\right)^{\phi - 1} - \gamma_\theta. \end{aligned}$$

However, this expression is identically zero for all θ from (3), which is the zero profit condition for the highest-priced sender in sector θ. It remains to show that there is no candidate solution other than the market equilibrium where the partial derivatives are equalized for the same $N = N^e$. This is impossible because $\frac{\partial^2 W}{\partial n_\theta^2} < 0$: if one n_θ were lower, then at least one other must be higher to not violate the constraint. But then the first of these would involve $\frac{\partial W}{\partial n_\theta} > 0$, and the second $\frac{\partial W}{\partial n_\theta} < 0$, which means they cannot be equalized.

To prove the second part (excessive messages), recall that $\frac{dW}{dn_\theta} = \frac{\partial W}{\partial n_\theta} + \frac{\partial W}{\partial N}$, from above, and we have just argued that $\frac{\partial W(n^e,N^e)}{\partial n_\theta} = 0$ for all θ. Then we have that $\frac{dW(n^e,N^e)}{dn_\theta} = \frac{\partial W(n^e,N^e)}{\partial N}$. From (12), we have $\frac{\partial W(n,N)}{\partial N} = \sum_{\theta=1}^{\Theta} (b_\theta - c_\theta)\, q_\theta \frac{d\mathbb{P}_\theta(n_\theta,N)}{dN}$ and this is strictly negative. ■

The first key feature here is the one that generates the optimality result. This is that the marginal change in the choice probability holding fixed the total number of messages, $\frac{\partial \mathbb{P}_\theta(n_\theta,N)}{\partial n_\theta}$, which is instrumental in the social problem, is equal to $\mathbb{Q}_\theta$, the probability the highest-priced firm makes a sale in the private problem. The equivalence holds because the probability that an extra message is examined and nothing else was examined from the sector both reflects its social contribution and the private incentive for sending it. In neither case are we concerned about it crowding out other messages from the sector: in the private case there any other message takes precedence by dint of its lower price and in the public case again only the extra likelihood of being examined counts.

The presentation above underscores the main problem with the market equilibrium: although the allocation is optimal across sectors given the total message volume, the *overall volume is excessive*. This is seen clearly from the condition we just argued, namely that $\frac{\partial W(n^e,N^e)}{\partial n_\theta} = 0$ (i.e., evaluated at the equilibrium N), while $\frac{dW(n^e,N^e)}{dn_\theta} < 0$. However, while optimal and private incentives are aligned in terms of allocation, the private choice ignores the message crowding externality on all other sectors, which is measured by $\frac{\partial W(n^e,N^e)}{\partial N}$. This means excessive messages are sent.

5.1 Taxing Transmission

The results above suggest that taxing transmission will raise welfare. The next Proposition clarifies in which sense.

Proposition 9 *A percentage transmission tax increases welfare. Price dispersion remains unchanged, as does the fraction of messages sent per sector, while the number of messages per sector (and therefore the total) go down in proportion to the percentage tax.*

Proof. A common percentage tax, τ, on transmission raises each γ_θ to $\gamma_\theta(1+\tau)$ and so reduces each π_θ proportionately to $\frac{\pi_\theta}{1+\tau}$. From (5), such a common tax means $N(1+\tau)$ is constant: equivalently, the original N falls to $\frac{N}{1+\tau}$. Recall $\frac{n_\theta}{N} = 1 - \left(\frac{1}{\phi}\frac{N}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$ from (4). Since the ratio $\frac{N}{\pi_\theta}$ (on the RHS) is unaltered by the tax, then so is the ratio $\frac{n_\theta}{N}$ (on the LHS). Likewise, since $\frac{N}{\pi_\theta}$ is unchanged, the price support and the cumulative price distribution remain unaltered too. Consumer welfare therefore remains unchanged, profits remain zero, and so welfare rises by the amount of tax raised. ■

The economics of raising transmission rates are the economics of rent dissipation. Doubling the cost in each sector simply halves the number of ads sent per sector. The intuition comes from the fact that both N and then n_θ

are homogeneous of degree minus one. The sector choice probabilities (n_θ/N) are then homogeneous of degree zero in the percentage transmission tax. The advertised price distribution, $F(p,\theta)$ is then also independent of the tax rate.

We just showed that percentage (across the board) cost increases have no effect locally on total transmission costs as borne by senders, due to adjustment to the zero profit equilibrium. If tax revenues were discarded, a tax would have no effect on welfare. Any tax not lost in the collection is therefore a social gain, and gets transferred purely from costs. Since profits are zero, consumers are just as well off since they face the same situation (same distributions,but just fewer overall messages). The tax is therefore raised *without deadweight loss*.

We proved above that the base allocation of messages was optimal for the equilibrium message volume, N^e. The equal percentage tax scales back messages proportionately. However, this does NOT mean that the scaled-back message volumes are also optimal for the new (given) total. Indeed, in Proposition 8 we have the partial welfare derivative

$$\frac{\partial W(n,N)}{\partial n_\theta} = (b_\theta - c_\theta)\, q_\theta \frac{\phi}{N}\left(1-\frac{n_\theta}{N}\right)^{\phi-1} - \gamma_\theta,$$

and this expression still holds in the presence of a tax (although the arguments are proportionately smaller). These partial derivatives are still to be equalized across sectors at any optimal allocation for given N. However, the market equilibrium condition in the presence of a tax becomes $(b_\theta - c_\theta)\, q_\theta \frac{\phi}{N}\left(1-\frac{n_\theta}{N}\right)^{\phi-1} = \gamma_\theta(1+\tau)$. Substituting,

$$\frac{\partial W(n,N)}{\partial n_\theta} = \tau\gamma_\theta. \tag{13}$$

This means that the allocation is constrained optimal either if $\tau = 0$ (where we evaluated the earlier welfare derivative), or if all the γ_θ are equal. Otherwise, ramping up the transmission cost with a tax causes an allocative distortion: from (13), the higher-cost messages ought to be provided more (and the lower-cost ones less). This means that the cheaper messages tend to be overused in equilibrium (in the presence of the tax).[18] These are the ones associated with the most dissipation, ceteris paribus.

Note that although the tax does not effect choice probabilities, the fact that the allocation is no longer optimal means that the optimal tax (given a target N) is not a proportional one. The results above instead suggest that the optimal tax should fall more heavily on the cheaper message communications.

[18] There are similar results in Transportation. Consider one origin, one destination and user costs linear in the number of users (and different across roads). The (Wardrop) equilibrium and the social optimum usage coincide if the cost when there is no one on the road (the free flow cost) is the same across roads. Let the cost per person per road be written $a_i + b_i n_i$: equivalence holds when the a_i are all the same. (Think of a simple diagram with two sectors: clearly the MSC cut at same allocations as the ASC if the a's are equal and ASC's are linear.) Otherwise, "the shortest route tends to be overused at equilibrium." Likewise here, the cheapest communication messages will be overused.

5.2 Crowding out by lower qualities?

Proposition 8 suggests there is not a sense in which the lower profit sectors in particular hurt the bigger ones, or vice versa, at the equilibrium. All sectors are in excess, but no group should be singled out. However, the discussion at the end of the last subsection indicates that if they were all scaled back to half size, by the expedient of a 100% tax on transmission say, then the high profit sectors would be overrepresented.[19] This at least suggests that the higher profit alternatives might be over-represented in the population of messages (in the sense that they ought to be scaled back more than proportionately).

This question leads us to ask whether an improvement in a sector - say a reduction in the sector's sending cost - can reduce welfare. As we shall show, such a reduction cannot hurt if all sectors are roughly similar, but it can if they are sufficiently asymmetric and a low-profit sector improves (becomes viable, say).

Recall first the welfare function is $W(n,N) = \sum_{\theta=1}^{\Theta} [(b_\theta - c_\theta) q_\theta \mathbb{P}_\theta (n_\theta, N) - \gamma_\theta n_\theta]$. Hence

$$\begin{aligned} \frac{dW}{d\gamma_i} &= -n_i + \sum_{\theta=1}^{\Theta} \frac{dn_\theta}{d\gamma_i}\frac{\partial W}{\partial n_\theta} + \frac{\partial W}{\partial N}\frac{\partial N}{\partial \gamma_i} \\ &= -n_i + \frac{\partial W}{\partial N}\frac{dN}{d\gamma_i} \end{aligned}$$

since each $\frac{\partial W}{\partial n_\theta} = 0$ in the neighborhood of equilibrium. The resulting expression for $\frac{dW}{d\gamma_i}$ indicates that there is a trade-off. Now, since $\frac{\partial W}{\partial N} = \sum_{\theta=1}^{\Theta} (b_\theta - c_\theta) q_\theta \frac{\partial \mathbb{P}_\theta(n_\theta,N)}{\partial N}$; and, since $\mathbb{P}_\theta(n_\theta, N) = 1 - \left(1 - \frac{n_\theta}{N}\right)^\phi$, then $\frac{\partial \mathbb{P}_\theta(n_\theta,N)}{\partial N} = -\frac{\phi}{N}\frac{n_\theta}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1} = -\frac{n_\theta}{N}\mathbb{Q}_\theta$. Recalling the equilibrium condition for the highest-priced sender entails $\mathbb{Q}_\theta = \frac{1}{\pi_\theta}$, then $\frac{\partial W}{\partial N} = -\sum_{\theta=1}^{\Theta} (b_\theta - c_\theta) q_\theta \frac{n_\theta}{N}\mathbb{Q}_\theta = -\sum_{\theta=1}^{\Theta} \frac{n_\theta}{N}\gamma_\theta$. The other desired term is $\frac{dN}{d\gamma_\theta}$. Recall first (from (5)) that $N = \phi \frac{(\Theta-1)^{\phi-1}}{\left(\sum_{\theta=1}^{\Theta}\left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right)^{\phi-1}}$; so, after manipulation, $\frac{dN}{d\gamma_\theta} = -N\frac{1}{\gamma_\theta}\frac{1}{\Theta}\frac{\chi_\theta}{\bar{\chi}}$, where we recall that $\bar{\chi}$ is the average value of $\chi_\theta = \left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$.

Pulling these expressions together, the derivative condition is:

[19] If half the messages were discarded across the board, then the marginal benefit of a message in a sector would be (retaining the original number values as arguments): $\frac{\partial W(n,N/2)}{\partial n_\theta} = (b_\theta - c_\theta) q_\theta \frac{2\phi}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1} - \gamma_\theta$. Since $(b_\theta - c_\theta) q_\theta \frac{\phi}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1} = \gamma_\theta$ at the original equilibrium, then $\frac{\partial W(n,N/2)}{\partial n_\theta} = \gamma_\theta$. This also indicates a benefit from scaling back the low-cost end relatively more.

$$\begin{aligned}\frac{dW}{d\gamma_i} &= -n_i + N\frac{1}{\gamma_i}\frac{1}{\Theta}\frac{\chi_i}{\bar{\chi}}\sum\nolimits_{\theta=1}^{\Theta}\frac{n_\theta}{N}\gamma_\theta \\ &= -n_i + \frac{1}{\gamma_i}\frac{1}{\Theta}\frac{\chi_i}{\bar{\chi}}\sum\nolimits_{\theta=1}^{\Theta} n_\theta\gamma_\theta\end{aligned}$$

Note first that under symmetry; then $\frac{dW}{d\gamma_i} = -n_i + n_i = 0$. This means that a rise in one sector's transmission costs has no effect at the margin under symmetry. To deal with asymmetric cases, it is helpful to rewrite the above expression as

$$\begin{aligned}\frac{dW}{d\gamma_i} &\overset{s}{=} -\frac{n_i\gamma_i\Theta}{\sum\nolimits_{\theta=1}^{\Theta} n_\theta\gamma_\theta} + \frac{\chi_i}{\bar{\chi}} \\ &= -\frac{\Gamma_i}{\bar{\Gamma}} + \frac{\chi_i}{\bar{\chi}}\end{aligned}$$

where the s label denotes that the derivative has the sign of the expression, and where Γ_i is the aggregate sector transmission cost. A marginal sector has Γ_i close to zero because it delivers few messages: if the sector has higher costs per message than average, then its π is low, so its χ is high. For a marginal sector, the first term is close to zero (because the volume of messages is vanishing), and the second term dominates: the poor sector's raise in costs – which effectively can bring about its demise – is socially beneficial. In that sense, it is the poorly performing products that are worst in the market place. This holds even though they have a small foothold: one might have otherwise suspected the high-profit products because they are responsible for the most crowding. It is important to note that the result of the previous subsection, which pointed the finger at the low-cost products as being over-represented, was derived under the assumption that all messages were scaled back proportionately (equivalently, the tax was a proportional one). Here it is an increase in the cost of a message that is being increased.

6 Distractions

Several of the strong properties in the normative analysis above relied critically on the homogeneity property of the numbers of messages sent. One natural way to relax this property is to introduce further competition for attention.

Think of consumers as having a limited amount of time, or a limited attention span. They cannot process all the information coming at them. Jostling with the price of MicroSoft Word or a supermarket flyer for pork chops is an email from a Dean or a crying child. We model this outside competition for attention as further "distractions" to attention. Formally, this means there are n_0 other messages (or activities) which compete for attention along with the messages sent from the advertising sectors. Hence now we have $N = \sum\nolimits_{\theta=0}^{\Theta} n_\theta$. We

will further associate an exogenous social value π_0 to each message (or activity) examined from the outside sector, and we assume that this value accrues on EACH such message examined.

This amendment relaxes some of the stronger properties of the equilibrium configuration, but retains other key ones, most notably that the market equilibrium is still optimal given N^e. However, raising τ in all sectors (except the distraction sector, which in that sense can be viewed as an untaxed sector) no longer causes the price spread to remain unchanged for all sectors. The key property before was that $N/\pi(1+\tau)$ was independent of τ. Now that is no longer true, and some sectors get evicted as tax rates rise.

6.1 Message volume with distractions

With distractions, it is still true that each sector's message share is $\frac{n(\theta)}{N} = \left(1 - \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right)$, $\theta = 1, ..., \Theta$ (see (1)). However, to find the total number of messages, N, now means adding in the outside sector, so the earlier CES form is amended to yield the implicit form:

$$\begin{aligned} N &= \sum\nolimits_{\theta=0}^{\Theta} n(\theta) \\ &= n_0 + N \sum\nolimits_{\theta=1}^{\Theta} \left(1 - \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right). \end{aligned}$$

Writing this out, we have

$$n_0 + N(\Theta - 1) = N^{\frac{\phi}{\phi-1}} \sum\nolimits_{\theta=1}^{\Theta} \left(\left(\frac{1}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right). \tag{14}$$

The LHS is linear in N (with a positive intercept), and the RHS is convex (and starts at 0), so that there is one and only one intersection with $N > 0$. Hence there is a unique solution $N > 0$. The comparative static properties of the equilibrium are quite simple. For example, a higher value of n_0 leads to a lower N, and n_θ falls in all other sectors.

6.2 Welfare analysis: right allocation of N but too many messages.

The welfare function is now written as $W = \sum\nolimits_{\theta=1}^{\Theta} \left[(b_\theta - c_\theta) q_\theta \mathbb{P}_\theta - \gamma_\theta n(\theta)\right] + n_0 \pi_0 \frac{\phi}{N}$, where π_0 denotes the net social benefit to the distraction, and n_0 distractions vie for the attention span of ϕ given N total competitors. The result of Proposition 8 with $n_0 > 0$ still holds true: with a distraction, the equilibrium allocation is still constrained optimal.

The proof follows the lines of the earlier one: again, for the active message-sending sectors, all the marginal benefits are equalized, 1...Θ. For any given N,

the partial derivative marginal benefit expressions (which are to be equalized across all sectors in the second-best problem of choosing the optimal allocation of N messages) are the same as those given before, and hence the constrained optimal problem still has the "right" allocation of the messages across the Θ sectors.

6.3 tax

Consider a tax on all sectors, except the "untaxed" sector, n_0. The welfare function now becomes

$$W = \sum_{\theta=1}^{\Theta} \left[(b_\theta - c_\theta) \mathbb{P}_\theta - \gamma_\theta (1+\tau) n_\theta \right] + n_0 \pi_0 \frac{\phi}{N} + \sum_{\theta=1}^{\Theta} \gamma_\theta \tau n_\theta.$$ [20]

The effect of a tax is given as $\frac{dW}{d\tau} = \sum_{\theta=1}^{\Theta} \frac{\partial W}{\partial n_\theta} \frac{dn_\theta}{d\tau} + \frac{\partial W}{\partial N} \frac{dN}{d\tau}$. Now, evaluating at $\tau = 0$, we have the result that the equilibrium entails the optimal allocation, $\frac{\partial W}{\partial n_\theta} = 0$, where the zero comes from the zero profit condition, as seen before. Hence, $\frac{dW}{d\tau} = \frac{\partial W}{\partial N} \frac{dN}{d\tau}$; and $\frac{\partial W}{\partial N} < 0$ as argued before, since each $\mathbb{P}_\theta$ term is decreasing in N (it becomes harder to find a sector) and the additional term, $n_0 \pi_0 \frac{\phi}{N}$, is decreasing as long as $\pi_0 > 0$. Since $\frac{dN}{d\tau} < 0$, welfare increases from the tax.

7 References

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[20] To find $N(\tau)$, and hence $n_\theta(\tau)$, use the previous expressions and note that $n_0 + N(\Theta - 1) = N^{\frac{\phi}{\phi-1}} (1+\tau)^{\frac{1}{\phi-1}} \sum_{\theta=1}^{\Theta} \left(\left(\frac{1}{\phi} \frac{1}{\pi_\theta} \right)^{\frac{1}{\phi-1}} \right)$, so that total messages go down.

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